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Lyapunov exponents of impact oscillators with Hertz's and Newton's contact models

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ABSTRACT

In this paper, investigations of a harmonically excited one-degree-of-freedom mechanical system having an amplitude constraint are presented. The contact between the oscillated mass and the barrier is modeled by Hertz's law with a non-linear damping as well as by Newton's law. The influence of the frequency of excitation force on the system's behavior is studied in a wide range of the control parameter by determining and analyzing the corresponding spectra of Lyapunov exponents. The dynamical behaviors of two systems with impacts: a system with Hertz's undamped impacts and a system with perfectly elastic hard impacts, which are equivalent in the sense of the same rate of impact energy dissipation, are compared and strong qualitative and quantitative similarities are observed. As an application example, a simple cantilever beam system with impacts is considered and the combined effects of the nonlinearities due to beam deflection and impacts of Hertz's as well as Newton's types are investigated.

1. Introduction

The calculation of Lyapunov exponents is one of fundamental elements in analysis of nonlinear dissipative systems with a finite number of degrees of freedom. They are numerical characteristics that allow for qualitative and quantitative evaluation of the system dynamics. These quantities are strictly connected to such measures of chaos as the Kolmogorov entropy and dimension of the dynamical system. The theoretical foundations for existence and uniqueness of Lyapunov exponents have been presented by Oseledec [1]. A spectrum of Lyapunov exponents characterizes the medium expansion of a small subset in the phase space along the trajectory. To identify the character of the system dynamics, it is usually enough to know the sign of the largest Lyapunov exponent —its non-positiveness renders the regularity of the system motion, whereas its positiveness—proves the chaotic character of the solution. The value of the largest Lyapunov exponent describes the rate of the mean exponential convergence or divergence of adjacent trajectories on the attractor.

The first method to calculate the whole spectrum of Lyapunov exponents was presented independently by Benettin et al. [2] and Shimada and Nagashima [3]. In the literature, there are two classical approaches towards determination of Lyapunov exponents for smooth systems with known equations of motion. In the first one, (see, for instance, [4]), an evolution of infinitesimal vectors of distortions in the trajectory under consideration is described by means of linearization of the vector field. The second approach (see, for example, [5]) consists in a substitution of the continuous system by its discrete counterpart, for instance, by applying Poincare maps, and a consideration of the linearization of the discrete map. Some alternative methods proposed by Stefanski (see, for instance, [6]) and Dabrowski (see, [7]) allow for determination of the largest Lyapunov exponent on the basis of the synchronization phenomenon of pairs of identical systems and the derivative dot product of perturbation vector, respectively.

Methods that enable estimation of Lyapunov exponents from experimental time series, that is to say, in the case when the system of differential equations describing the behavior of the system is not available, are known as well. Their basis usually lies in a reconstruction of the state space with the delay method, introduced by Takens [8]. The first procedure of this kind for calculation of the largest Lyapunov exponent was given by Wolf et al. [9], whereas analogous algorithms for determination of the whole spectrum of Lyapunov exponents can be found in Parlitz [10], Sano and Sawada [11], and Yang and Wu [12].

In the literature, a few adaptations of classical methods for determination of Lyapunov exponents to the case of piecewise smooth systems can be found. Müller [13] (cf [14]) has shown that

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the conditions for transition of the system through the non-smoothness have their counterparts for the linearized system, due to which it is possible to determine Lyapunov exponents with a classical method of the Benettin et al. type [2]. A similar modification of the discrete method, based on the notion of local Nordmark maps [15], has been presented by Jin et al. [16]. A different approach with a smaller range of applications limited to piecewise linear systems, consists in an application of discontinuity maps ([17,18]) instead of Poincare maps.

An important part of the dynamical systems is represented by those systems whose motions take place in the presence of impacting interactions between the masses of the system (see [19,20]). The classic approach to study the collision process, called stereomechanical model of a collision [21] or hard collision model [22], uses the coefficient of restitution and the principle of conservation of momentum, and allows to determine the velocity of the bodies after the collision on the basis of knowledge of the velocity of the bodies before the collision. Taking into account the duration of the impact and the coefficient of restitution depending on the velocity, leads to models which more accurately describe the process of collision (see e.g. [23,24]). Such a process is similar to the collision with the stop with a certain vulnerability, and is called a soft collision model [22]. In this case, there is a choice of stops modeling. They can be linear (e.g., models of vibroimpact systems with clearance [25–28]) or nonlinear (e.g., Hertz's models [21,29,30]), elastic or elastic-damping constructions. A comprehensive survey of the current knowledge about systems with impacts has been made by Ibrahim [31].

The above-described methods of deriving Lyapunov exponents have been applied to impact systems with rigid stops ([32–36]), except for [37], in which Lyapunov exponents have been calculated with the method of impact maps for piecewise linear one-degree-of-freedom systems with one-sided impacts.

In this paper, we deal with a one-degree-of-freedom linear oscillator with impacts modeled with soft nonlinear elastic structures (Hertz's contact model [21]), soft nonlinear elastic-damping structures (Hertz's damp contact model [30]) as well as Newton's law of contact, which besides its own interest, aims at representing an impacting cantilever beam system. The main objective is to analyze qualitatively and quantitatively the influence of the frequency of excitation force on the system's behavior in the case of these three contacts models as well as to compare the resulting responses. To this aim, we adapt the Müller's approach and determine numerically the spectra of Lyapunov exponents. The results obtained are consistent with the corresponding bifurcation diagrams.

The presented study shows that the knowledge of Lyapunov exponents enables more detailed analysis of the system's behavior in comparison to other tools, e.g., Poincare maps or bifurcation diagrams. In particular, it allows to identify some phenomena which have not been reported on the basis of bifurcation diagrams, like some periodic orbits not identified in the study by Pust and Peterka [30]. Furthermore, we show that Lyapunov exponents can provide a tool for not only qualitative (cf. [29]) but also quantitative comparison of different systems with impacts. The presented comparison of dynamical behaviors of a system with Hertz type undamped collisions of relatively small values of stiffness and a system with perfectly elastic hard collisions revealed their good qualitative and quantitative agreement. This agreement manifests itself in the appearance, for almost the same values of the excitation force, of the chaotic motions with almost identical values of the Lyapunov exponents corresponding to both the collisions models, as well as in the existence, in a wide range of the excitation force, of periodic motions with impacts, for which the corresponding Lyapunov exponents are very close to each other. In particular, this is the case when the two systems begin to come into collisions with low velocity impacts, causing instabilities of grazing-type.

From the mechanical engineering point of view, our results apply to a simple cantilever beam system with impacts, which is commonly used as an element of engineering design. However, if the beams are parts of a larger system, significant errors in the dynamical responses can result from neglecting even small nonlinearities. The cumulative effect of the nonlinearity associated with the beam deflection and the nonlinearity due to impact model with clearance and linear spring was examined by Emans et al. [38] and Lin et al. [39]. We extend these studies to two other impact models. A comparison of dynamic responses of simple linear and nonlinear beam systems with impacts of Hertz's and Newton's type revealed their qualitative differences for physically realistic parameters.

This paper is organized as follows. Mathematical models of the considered system are introduced in Section 2. In Sections 3 and 4, the classical method for Lyapunov's exponents determination as well as its modification for systems with singularities are briefly described. Analysis of a harmonically excited one-degree-of-freedom impact oscillator with two Hertz's models of contact carried out with the help of the corresponding spectra of Lyapunov's exponents as well as a comparison of dynamics of a system with perfectly elastic hard impacts and an equivalent, in the sense of the same rate of impact energy dissipation, system with Hertz's impacts, are presented in Section 5. In Section 6, the cumulative effect of different type nonlinearities on cantilever beam responses is investigated. The conclusions are formulated in Section 7.

2. Mathematical model of the system

The system under consideration consists of a linear oscillator with mass m, coefficient of viscous damping c and spring stiffness coefficients k and k_e, presented in Fig. 1. The oscillator can be under either external kinematic excitation (Fig. 1a) or external forcing (Fig. 1b). In the first case the upper end of the spring k_e moves harmonically with the assigned amplitude a and frequency ω. In the second case, the harmonic force of the assigned amplitude F and frequency ω acts on the oscillator. When the oscillators are in their static equilibrium positions, the distance between their impacting surfaces and the unmovable fender is ρ. The motion of the oscillators around their static equilibrium positions is described by coordinate x.

The equations of impactless motion of the above-described systems are as follows:

– for the oscillator shown in Fig. 1a

$$m\frac{d^2x}{dt^2}+c\frac{dx}{dt}+(k+k_e)x=k_e a\cos\omega t, \quad (1a)$$

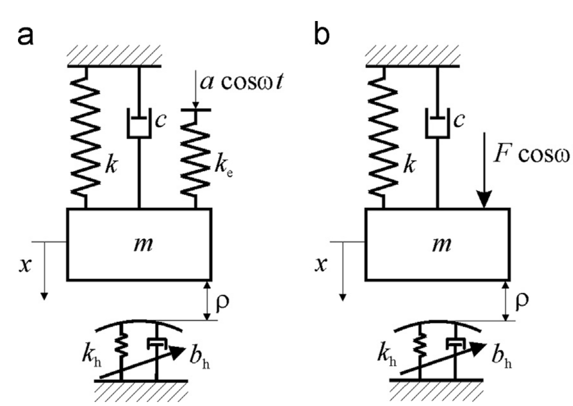

Fig. 1. Impacting oscillators with two types of external excitation.