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Dynamic instability of an elastic solid sliding against a functionally graded material coated half-plane

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ABSTRACT

Elastic dynamics in the stability of an elastic solid sliding against a functionally graded material (FGM) coated half-plane is investigated by examining the stability of elastic waves caused by the perturbation. The material properties of the FGM coating vary exponentially along the thickness direction. The effects of the gradient index, friction coefficient and sliding speed for various material combinations on the dynamic instability are discussed in detail. The transverse normal stresses in both coating and homogeneous half-plane are calculated and the effect of the graded coating on the stress distribution is also discussed. It is shown that the FGM coating can be used to modify sliding stabilities and control the interfacial tensile stress, and thus reduce the possibility of the interfacial contact failure simultaneously.

1. Introduction

Sliding friction is one of the oldest areas of research [1]. In particular, the frictional motion is often unstable and gives rise to non-uniformities, noise and vibrations, ranging from nano-tribology [2] to "squeal" or "hot-spotting" in brakes or clutches [3,4], up to the scales of sliding of tectonic plates in earthquakes [5–7]. Martins et al. [8] investigated dynamic instabilities of the steady frictional sliding of a linear elastic or viscoelastic half-plane compressed against a rigid plane which moved with a prescribed non-vanishing tangential speed. These instabilities were thought to play a role in Schallamach waves [9]. Adams [10] found that the steady sliding of two elastic half-planes was also dynamically unstable. Steady-state sliding can give rise to a dynamic instability in the form of self-excited motion. Later, Adams [11] used a simple beam-on-elastic-foundation model to investigate instabilities caused by the sliding of a rough surface against a smooth surface. He observed that the instability would eventually lead to either partial loss of contact or to stick-slip motion. Adams [12] indeed showed that stick-slip motion at the interface can exist with a constant friction coefficient. Wang et al. [13] investigated the slip waves propagating along an interface between two anisotropic solids in the frictional sliding contact with local stick-slip. Another type of instability is the frictional heating induced instability. The simple 1D and 2D models were studied by Afferrante et al. [14] and Afferrante and Ciavarella [15–17] by considering an elastic layer sliding against a rigid wall. They found that thermal effects can render unstable the otherwise neutrally stable natural elastodynamic modes of system, giving rise to a new family of instability, which is called TEDI. Later, Afferrante and Ciavarella [18] considered the general case of two sliding elastic half-planes, again finding the general family of instability TEDI class.

In some frictional contact problems, a thin tribological layer between two bodies was introduced. Adams [19] discussed the effect of surface layers on dynamic instabilities in the sliding of two elastic half-planes. For the case of a rough and stiff surface topography of the layer, Slavic et al. [20] provided a model to describe roughness-based vibrations of the sliding system.

In fact, the conventional homogeneous layered structure may cause the "interface-problem" by the abrupt change in the material properties at the interface, which results in the stress concentration, degraded bonding strength and consequently susceptibility to interface failure under the heavy contact loading. Furthermore, the conventional homogeneous layered structure may affect the sliding stabilities [19]. FGMs possess properties that vary gradually with location within the material. Used as coatings and interfacial layers, they can reduce the magnitude of residual and thermal stresses, mitigate stress concentration and increase fracture toughness [21]. In order to improve sliding stabilities as well as alleviate the "interface-problem", we try to introduce the functionally graded materials (FGMs) as the coating. Therefore, the main purpose of the present investigation is to determine the effect of an FGM coating on dynamic instabilities in the sliding of two elastic half-planes.

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Elastic dynamic instability of an elastic solid sliding against an FGM coated half-plane is investigated in this paper by examining the stability of elastic waves caused by a perturbation. The material properties of the FGM layer are assumed varying exponentially along the thickness direction. A parametric study is conducted to highlight the effects of the gradient index, fiction coefficient and sliding speed for various material combinations on the dynamic instability. In addition, the transverse normal stresses varying in the direction of the depth for both FGM coated and homogeneous coated structures are determined.

2. Problem description

We consider a homogeneous elastic half-plane sliding against an FGM coated half-plane under the normal force $\hat{P}_0$ and tangential force $\hat{Q}_0$, see Fig. 1. Friction follows the Coulomb friction law, i.e., $\hat{Q}_0 = f\hat{P}_0$ with f being the friction coefficient. The lower and upper half-planes are homogeneous with the shear moduli μ_1, μ_2 and mass densities ρ_1, ρ_2, respectively. It is assumed that the lower and upper half-planes move with constant speeds $\hat{V}_1$, and $\hat{V}_2$, respectively. That is to say, two half-planes slide with a relative speed $\hat{V}_0 = \hat{V}_2 - \hat{V}_1$. The problem is formulated in a coordinate system $(\hat{x}, \hat{y})$ moving to the right with a constant speed $\hat{V}$ where $\hat{V}$ is not necessarily equal to $\hat{V}_0$. The FGM coating is perfectly bonded to the lower half-plane, and its properties vary along the thickness direction according to the exponential function

$$\mu(\hat{y}) = \mu_0 e^{\alpha\hat{y}}, \quad \rho(\hat{y}) = \rho_0 e^{\alpha\hat{y}}, \tag{1}$$

where $\hat{\alpha}$ is the gradient index of the FGM coating; and μ_0 and ρ_0 are the shear modulus and mass density of the bottom $(\hat{y} = 0)$ of the FGM coating, respectively.

In this paper, we will examine the stability of the above sliding system by assuming the system to be disturbed with a very small perturbation. Then an elastic wave will be generated and propagate through the system. If the elastic wave decays with the time, then the sliding is stable; otherwise if the elastic wave gains with the time, then the sliding is unstable and will lead to the sliding instability. This dynamic instability can eventually give rise to the partial separation or the stick-slip regions.

By considering the elastic wave caused by the small perturbation, the displacement and stress may be decomposed into two parts, i.e.,

$$\hat{\mathbf{u}}^t(\hat{x}, \hat{y}, \hat{t}) = \hat{\mathbf{u}}^*(\hat{x}, \hat{y}) + \hat{\mathbf{u}}(\hat{x}, \hat{y}, \hat{t}), \tag{2}$$

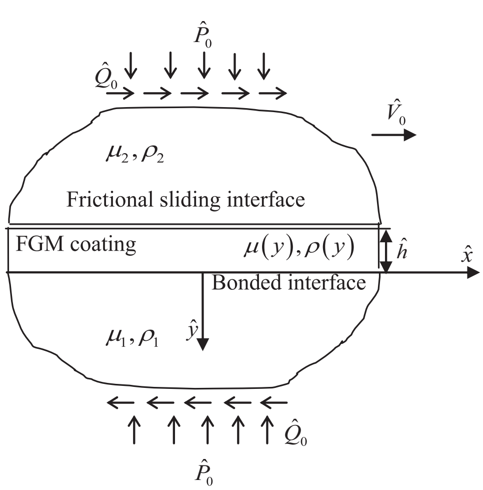

Fig. 1. A homogeneous elastic half-plane sliding against an FGM coated half-plane.

$$\hat{\boldsymbol{\sigma}}^t(\hat{x}, \hat{y}, \hat{t}) = \hat{\boldsymbol{\sigma}}^*(\hat{x}, \hat{y}) + \hat{\boldsymbol{\sigma}}(\hat{x}, \hat{y}, \hat{t}), \tag{3}$$

where $\hat{t}$ is time; and $\hat{\mathbf{u}}^*(\hat{x}, \hat{y})$ and $\hat{\boldsymbol{\sigma}}^*(\hat{x}, \hat{y})$ are quasi-static displacement and stress which are invariant in time and satisfy

$$\hat{\sigma}^*_{yy}\left(\hat{x}, -\hat{h}\right) = -\hat{P}_0, \quad \hat{\sigma}^*_{xy}\left(\hat{x}, -\hat{h}\right) = -\hat{Q}_0, \tag{4}$$

on the frictional sliding interface $(\hat{y} = -\hat{h})$. The second terms $\hat{\mathbf{u}}(\hat{x}, \hat{y}, \hat{t})$ and $\hat{\boldsymbol{\sigma}}(\hat{x}, \hat{y}, \hat{t})$ correspond to the elastic wave caused by the small perturbation and are the main concern in the present study. They need to satisfy the boundary conditions at the frictional sliding interface $\hat{y} = -\hat{h}$, i.e.,

$$\hat{\sigma}_{xy}\left(\hat{x}, -\hat{h}, \hat{t}\right) = f\hat{\sigma}_{yy}\left(\hat{x}, -\hat{h}, \hat{t}\right), \tag{5}$$

$$\left[\hat{u}_y\left(\hat{x}, -\hat{h}, \hat{t}\right)\right] = 0, \quad \left[\hat{\sigma}_{yy}\left(\hat{x}, -\hat{h}, \hat{t}\right)\right] = 0, \tag{6}$$

where $[\cdot]$ denotes the discontinuities of the interfacial displacement and stress components. In addition, the continuity conditions on the bonded interface $\hat{y} = 0$, are

$$[\hat{u}_x(\hat{x}, 0, \hat{t})] = 0, \quad [\hat{u}_y(\hat{x}, 0, \hat{t})] = 0, \tag{7}$$

$$[\hat{\sigma}_{yy}(\hat{x}, 0, \hat{t})] = 0, \quad [\hat{\sigma}_{xy}(\hat{x}, 0, \hat{t})] = 0. \tag{8}$$

It is noted that the contact pressure $\hat{P}$ must be non-negative and the direction of the slip velocity (V_s) at the sliding interface is the same as that of $\hat{V}_0$ when there is no stick-slip or stick-slip-separation motion at the sliding interface.

In the moving coordinate system, the wave motion equations for the linear homogeneous isotropic half-plane are given by

$$\beta^2\frac{\partial^2\hat{u}_x}{\partial\hat{x}^2} + \frac{\partial^2\hat{u}_x}{\partial\hat{y}^2} + (\beta^2 - 1)\frac{\partial^2\hat{u}_y}{\partial\hat{y}\,\partial\hat{x}} = \frac{\hat{\rho}}{\hat{\mu}}\left(\hat{V}^2\frac{\partial^2\hat{u}_x}{\partial\hat{x}^2} - 2\hat{V}\frac{\partial^2\hat{u}_x}{\partial\hat{x}\,\partial\hat{t}} + \frac{\partial^2\hat{u}_x}{\partial\hat{t}^2}\right), \tag{9}$$

$$\beta^2\frac{\partial^2\hat{u}_y}{\partial\hat{y}^2} + \frac{\partial^2\hat{u}_y}{\partial\hat{x}^2} + (\beta^2 - 1)\frac{\partial^2\hat{u}_x}{\partial\hat{y}\,\partial\hat{x}} = \frac{\hat{\rho}}{\hat{\mu}}\left(\hat{V}^2\frac{\partial^2\hat{u}_y}{\partial\hat{x}^2} - 2\hat{V}\frac{\partial^2\hat{u}_y}{\partial\hat{x}\,\partial\hat{t}} + \frac{\partial^2\hat{u}_y}{\partial\hat{t}^2}\right), \tag{10}$$

where $\hat{V} = \hat{V}_1$ or $\hat{V}_2$; and ν is Poisson's ratio. The wave motion equations in the FGM coating are

$$\frac{2(1-\nu_0)}{1-2\nu_0}\frac{\partial^2\hat{u}_x}{\partial\hat{x}^2} + \frac{\partial^2\hat{u}_x}{\partial\hat{y}^2} + \frac{1}{1-2\nu_0}\frac{\partial^2\hat{u}_y}{\partial\hat{x}\,\partial\hat{y}} + \alpha\left(\frac{\partial\hat{u}_x}{\partial\hat{y}} + \frac{\partial\hat{u}_y}{\partial\hat{x}}\right)$$
$$= \frac{\rho_0}{\mu_0}\left(\hat{V}^2\frac{\partial^2\hat{u}_x}{\partial\hat{x}^2} - 2\hat{V}\frac{\partial^2\hat{u}_x}{\partial\hat{x}\,\partial\hat{t}} + \frac{\partial^2\hat{u}_x}{\partial\hat{t}^2}\right), \tag{11}$$

$$\frac{2(1-\nu_0)}{1-2\nu_0}\frac{\partial^2\hat{u}_y}{\partial\hat{y}^2} + \frac{\partial^2\hat{u}_y}{\partial\hat{x}^2} + \frac{1}{1-2\nu_0}\frac{\partial^2\hat{u}_x}{\partial\hat{x}\,\partial\hat{y}} + \frac{2\alpha}{1-2\nu_0}\left[(1-\nu_0)\frac{\partial\hat{u}_y}{\partial\hat{y}} + \nu_0\frac{\partial\hat{u}_x}{\partial\hat{x}}\right]$$
$$= \frac{\rho_0}{\mu_0}\left(\hat{V}^2\frac{\partial^2\hat{u}_x}{\partial\hat{x}^2} - 2\hat{V}\frac{\partial^2\hat{u}_x}{\partial\hat{x}\,\partial\hat{t}} + \frac{\partial^2\hat{u}_x}{\partial\hat{t}^2}\right). \tag{12}$$

In the following, we will solve Eqs. (9)–(12) under the boundary conditions (5)–(8) and analyze the stability of the elastic wave.

3. Elastic wave fields

A convenient dimensionless formulation can be developed by defining the normalized quantities

$$x = \hat{x}/l, \quad y = \hat{y}/l, \quad t = \hat{t}\hat{c}_s/l, \quad V = \hat{V}/\hat{c}_s, \quad \hat{c}_s = \sqrt{\mu/\rho}, \tag{13a}$$

$$u_x(x, y, t) = \hat{u}_x(\hat{x}, \hat{y}, \hat{t})/l, \quad u_y(x, y, t) = \hat{u}_y(\hat{x}, \hat{y}, \hat{t})/l, \tag{13b}$$

where the characteristic length l is the dimensional wavelength. Then, the wave motion Eqs. (9) and (10) for the upper half-plane can be written in the dimensionless form as