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Direct identification of nonlinear damage behavior of composite materials using the constitutive equation gap method

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ABSTRACT

This paper aims at improving the reliability of the identification of parameters that govern a non-linear damage law for orthotropic materials, the non-linearity considered being due to the inherent damage induced by in-plane shear loading. The adopted identification technique is based on the minimization of the constitutive equation gap (CEG). It is built upon the quadratic deviation between the measured and computed strain field. The paper explains how an appropriate use of the CEG method can improve the identification of material model parameters from strain field measured by an optical technique of full-field measurement. The minimization problem is solved by the modified Levenberg–Marquardt method. In the present work, the experimental data are replaced by numerical ones provided by the finite element (FE) simulations. These latter are performed on an orthotropic glass/epoxy composite material. The obtained parameters based on these numerical data are considered as reference ones. A sensitivity analysis is proposed as an attempt to examine the influence of some parameters, among which the mesh refinement, the initial set of parameters, the experimental noise, and the orthotropy ratio. Two types of noise simulating the measurement errors are tested: a random noise simulated by a Gaussian white noise and a bias noise. The obtained results demonstrated the feasibility and applications of the proposed method. The use of the CEG method gives results of good quality.

1. Introduction

The determination of parameters governing a constitutive law is a challenge that becomes difficult when their number is significantly high. Such a case occurs when anisotropic or/and non-linear constitutive laws are considered. The usual approach consists in performing several tests like tensile ones and fitting the model with the experimental data. However, the number of tests increases with the number of parameters. Moreover, parasitic effects can disturb the stress field which is usually expected to be uniform when homogeneous tests are performed. These drawbacks can be avoided by performing heterogeneous tests on non-standard specimens. In that case, only one specimen is tested and the strain/stress field is heterogeneous. The use of full-field measurement techniques is extensively required for the characterization of materials. Constitutive law parameters can be potentially extracted through an identification strategy using kinematics fields obtained from one single coupon. Indeed, owing to the heterogeneous strain/stress fields given rise to a heterogeneous test (by 'heterogeneous', it is understood that all stress/strain components are present and non-uniform), in that case, the constitutive parameters are expected to be all involved in the response of the specimen. The development of such an approach then requires a relevant choice of specimen geometry, a full-field measurement technique as well as an identification strategy.

Several types of full-field measurement technique have been proposed and used to measure the displacement/strain field to identify the material properties [1–7]. Strain fields can be obtained by numerical differentiation of the above displacement fields with suitable algorithms [8] or directly, for instance with shearography [9,10], speckle shearing photography [11] or by moiré fringes shifting [12].

There are five main techniques likely to be used to solve the identification problem: the virtual field method [3,13–16], the FE model updating method [17–21], the reciprocity gap method [22,23], the equilibrium gap method [24,25], and the constitutive equation gap (CEG) method [26–29]. A review of these techniques is presented in [30] with a focus on noisy data in [5]. In the present work, an inverse approach based on CEG is adopted to identify the material

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parameters governing a non-linear behavior law for orthotropic composite materials.

The CEG was initially proposed in [31] (error in constitutive relation) to build admissible stress fields when the boundary conditions of the reference problem are known. The use of the CEG has led to interesting results concerning the estimation of global or local errors in FE simulations [32]. It should be noted that the CEG is developed in the framework of verification and validation of FE simulations, and it is adopted in this work in order to identify the material model parameters as in [28,33–35].

The identification method presented in the present paper is based on the minimization of an energy norm expressed through CEG. The latter constitutes an error estimator expressed as a function of the quadratic gap between computed and measured strain fields obtained by a full-field measurement technique. Here, FE based values are used instead of full field measurements. The identification is described under the constraint expressed by the gap between the computed and measured boundary conditions.

Once the CEG method is formulated, one has to find a behavior parameters set minimizing this quantity. The minimization problem is solved using the Levenberg–Marquardt method [36,37]. This first order minimization method requires the computation of a sensitivity matrix defined by the derivatives of different computed data components with respect to the material parameters to be identified. Indeed, the most important aspect of this minimization algorithm is the determination of the sensitivity matrix. A good choice for the sensitivity matrix may give rise to less iterations and hence to reduced computation times. In the open literature, several techniques can be used to determine the sensitivity matrix. Among them, the analytical method presented in [38] or the adjoint method [39]. A mixed technique based on the semi-analytical method using the finite difference and analytical calculation is used [40]. In this work, the finite difference method is used to compute the sensitivity matrix. This method requires a computation with small perturbation of each set of parameters. The FE code Abaqus® [41] is adopted in order to solve the direct problem. The implementation of such an approach then requires a relevant choice of specimen geometry, an experimental technique of kinematic field measurements as well as an identification strategy.

In this work, the material parameters governing a non-linear behavior damage for composite materials is identified using CEG method, the non-linearity considered being due to the damage inherent to the in-plane shear response. The mesoscale damage model proposed by Ladevèze [42] is considered. The latter is extensively used for predicting the initiation and growth of many forms of damage in composite materials [42]. Then, the objective is to examine the efficiency of the method to identify material parameters of the damage model. The experimental data are replaced by numerical ones provided by the finite element (FE) simulations. Reference parameters governing the constitutive model of an orthotropic glass/epoxy composite are considered to generate these data using the FE code Abaqus® [41]. Two heterogeneous geometries of specimens are chosen. The first geometry is an open-hole tensile test. The second is an asymmetric double notched geometry. In these configurations, the strain/stress fields are heterogeneous. The whole behavior law parameters are extracted simultaneously from only one numerical mechanical test (replacing the experimental database which are not available yet).

In the present paper, the CEG method is first recalled. Then, the inverse problem solution is detailed. Finally, a numerical analysis is performed to test the robustness and the stability of the identification procedure, among which the sensibility to mesh refinement, the influence of the initial parameters, the effect of noise and the influence of the orthotropy ratio. Since the measurement noise corrupting the considered displacement field is unavoidable, the stability of the identification procedure is assessed by introducing two types of data noises. The first one is a random error simulated by a Gaussian white noise, the second one is a bias error simulating a shift measurement error induced by an out-of plane displacement for instance.

The obtained identified parameters are in good agreement with their reference counterparts in the presence of noisy data. The obtained results demonstrate the feasibility and capability of the proposed method. This study showed that it is possible to identify material parameters governing a non-linear behavior law from one mechanical test inducing heterogeneous strain/stress fields. We prove that the use of the CEG method significantly improves the quality of the identification of material model parameters.

2. The constitutive equation gap method

2.1. General presentation

Let us consider a structure (Ω) consisting of an elastic material. The equations of the problem can be separated into two groups:

- *Admissibility conditions*: Kinematic constraints, equilibrium and compatibility equations;
- *Constitutive relations*:

$$\underline{\sigma} = \underline{\underline{\mathcal{C}}} : \underline{\varepsilon}(u) \tag{1}$$

where $\underline{\sigma}$ denotes the stress tensor, $\underline{\varepsilon}$ the strain tensor associated with the displacement u, and $\underline{\underline{\mathcal{C}}}$ the fourth order elasticity Hooke's tensor.

It is worth noting that constitutive relations are often the least reliable equations describing a mechanical problem. Accordingly, to solve the problem, one must find an approximate displacement/stress solution satisfying the most reliable equations: admissibility conditions. Namely, the pair of solution ($\underline{\sigma}$, u) must satisfy the kinematic constraints, the equilibrium and the compatibility equations.

Let $(u_{KA}, \underline{\sigma}_{SA})$ be a displacement–stress pair, where u_{KA} is a Kinematically Admissible displacement field ($\mathbb{KA}$) and $\underline{\sigma}_{SA}$ is a Statically Admissible stress field ($\mathbb{SA}$). If the constitutive relation (1) is satisfied, the pair $(u_{KA}, \underline{\sigma}_{SA})$ is the exact solution of the problem. Hence, one can write

$$\underline{\sigma}_{SA} = \underline{\underline{\mathcal{C}}} : \underline{\varepsilon}(u_{KA})$$
$$\mathcal{KA} = \{u | u = \mathcal{U} \ over \ \partial\Omega_u\}, \quad \mathcal{SA} = \{\sigma | div\ \sigma = 0; \sigma.n = \tau \ over \ \partial\Omega_\sigma\}$$
$$\partial\Omega = \partial\Omega_u \cup \partial\Omega_\sigma \tag{2}$$

Generally, the numerical approximation of this pair does not satisfy the constitutive relation (1), consequently one obtains $\underline{\sigma}_{SA} - \underline{\underline{\mathcal{C}}} : \underline{\varepsilon}(u_{KA}) \neq 0$, and then the pair $(u_{KA}, \underline{\sigma}_{SA})$ is only an approximate solution of the considered problem. The quantity:

$$\underline{\sigma}_{SA} - \underline{\underline{\mathcal{C}}} : \underline{\varepsilon}(u_{KA}) \tag{3}$$

is referred to as the CEG associated with the admissible pair $(u_{KA}, \underline{\sigma}_{SA})$ at each point in the whole structure (Ω) [31]. This quantity is chosen to evaluate the quality of the pair $(u_{KA}, \underline{\sigma}_{SA})$ and it allows quantifying its variation in the exact solution in terms of the constitutive relation. The CEG can be defined by the error e_{CEG}:

$$e_{CEG} = \| \underline{\sigma}_{SA} - \underline{\underline{\mathcal{C}}} : \underline{\varepsilon}(u_{KA}) \|_\Omega \tag{4}$$

where

$$\|\bullet\|_\Omega^2 = \int_\Omega (\bullet : \underline{\underline{\mathcal{C}}}^{-1} : \bullet)\, d\Omega \tag{5}$$