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Influence of strain hardening on bending moment–axial force interaction

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ABSTRACT

This article derives bending moment–axial force (M–P) interaction curves for mild steel by considering elastic–plastic and strain hardening idealisations with linear and parabolic strain hardening characteristics. The interaction relations can predict strains, which is not possible in a rigid, perfectly plastic idealisation. The parameters required are obtained from a standard uniaxial tension test and the procedure can be used for many other materials.

1. Introduction

Structural designers are often required to estimate the failure load of structural members for which they employ numerical techniques, such as the finite element method, but the analysis up to failure with large displacements and strains is usually difficult. Continuum damage mechanics has been used recently [1,2] for predicting the static and dynamic failure of beams, but the method requires the values for several parameters, some of which are difficult to obtain.

An elastic–plastic model has been used, for example, to study the theoretical anomalous dynamic response of beams [3,4] and plates [5] for a short pulse loading causing small deflections. Another simpler and more attractive option for some problems is to carry out a rigid perfectly plastic analysis [6], the accuracy of which has been compared with the predictions of an elastic–plastic material [7,8]. However, a rigid, perfectly plastic analysis does not predict strains so that it is difficult to study failure unless some assumptions are made to overcome this difficulty.

Zyczkowski [9] presented a comprehensive review of theoretical and experimental papers concerning combined plastic loading of sections having different shapes. Many investigators have developed interaction curves or carried out interaction studies pertaining to the combined action of different stress resultants but the studies were mostly for elastic or elastic perfectly plastic cases [10–14]. Whereas, the interaction studies up to the ultimate capacity are either experimental or numerical [15,16].

In the present paper, kinematically admissible interaction curves for the simultaneous action of bending moment and an axial force on a rectangular section have been developed for elastic–plastic and strain-hardening material idealisations. These curves may be used for the failure analysis of structural elements.

2. Stress–strain diagram

The stress–strain diagram for mild steel is idealised as bilinear for small strains, whereas, two models – linear and parabolic – are used for strain-hardening (Fig. 1). Direct tensile test results for a mild steel specimen t036 [2] are shown in Fig. 1. Thus, there are three zones in the idealised diagram: elastic zone from $k=0$ to 1; yield zone without any strain-hardening from $k=1$ to k_1; and the strain-hardening zone from $k=k_1$ to k_2, where, $k\varepsilon_y$ is the strain. The stress in the strain-hardening range, σ_d, at any strain, $\varepsilon=k\varepsilon_y$ ($k_1 \le k \le k_2$), can be obtained from the following relations:

$$(\sigma_d-\sigma_{yd})=(\sigma_{ud}-\sigma_{yd})m \quad \text{for linear-hardening} \tag{1}$$

$$(\sigma_d-\sigma_{yd})=(\sigma_{ud}-\sigma_{yd})m(2-m) \quad \text{for parabolic-hardening} \tag{2}$$

where

$$m=\left(\frac{\varepsilon-\varepsilon_h}{\varepsilon_u-\varepsilon_h}\right)=\left(\frac{k-k_1}{k_2-k_1}\right) \tag{3}$$

The suffix d in the above expressions has been used to indicate dynamic values. The stress–strain curve can be used for high strength steel by substituting $k_1=1$ and many other materials can

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Nomenclature

a, b, c, C	parameters
A, q	Cowper–Symonds' parameters
B	width of rectangular section
E	modulus of elasticity
F	parameter for exponential model
G	shear modulus
h	H_1/H
H	depth of rectangular section
H_1	distance of neutral axis (NA) from extreme compression fibre
L	half span of beam
m, m'	parameters
M	bending moment at the section
$\overline{M}$	(M/M_{yd})=shape factor when yield stress is σ_{yd}
$\overline{M}'$	(M/M'_{yd})=shape factor when yield stress is σ'_{ud}
$\overline{M}_u$	(M_u/M_{yd})=ultimate shape factor
M_{yd}	dynamic yield moment$=\frac{1}{6}\sigma_{yd}BH^2$
M'_{yd}	$\frac{2}{3}M_{ud}=\frac{1}{6}\sigma'_{ud}BH^2$
M_{ud}	dynamic ultimate moment
n	parameter for exponential model
P	axial force on the section
P_{yd}	yield force$=\sigma_{yd}BH$
P_{ud}	ultimate force$=\sigma_{ud}BH$
$\overline{P}$	P/P_{yd}
$\overline{P}_u$	P/P_{ud}
R	radius of curvature
s	$((\sigma_{ud}/\sigma_{yd})-1)$
V	striking velocity
$\overline{y}$	distance

Greek symbols

α	parameter
ε	normal strain
ε_y	yield strain
$\varepsilon_m=k\varepsilon_y$	strain in extreme fibre
$\varepsilon_h=k_1\varepsilon_y$	strain corresponding to end of yielding and beginning of strain-hardening
$\varepsilon_u=k_2\varepsilon_y$	ultimate strain
$\dot{\varepsilon}$	strain rate
σ	normal stress
σ_{ys}	static yield stress
σ_{yd}	dynamic yield stress
σ_{us}	static ultimate stress
σ_{ud}	dynamic ultimate stress
σ'_{yd}	modified ultimate dynamic stress for plastic bending $=(4M_{ud}/BH^2)$

Subscripts

a	axial
c	collapse
d	dynamic
e	elastic
h	beginning of hardening
s	static
u	ultimate
y	yield

be easily represented by these equations for different values of the parameters.

The exponential model [17,18] is another model, which is used for idealising the stress–strain relation:

$$\sigma = E\varepsilon + F\varepsilon^{1/n} \tag{4}$$

where, E is the modulus of elasticity and the other parameters, F and n, can be found by fitting a known stress–strain curve, as shown in Fig. 1 for specimen t036 [2]. In the model shown in the figure, the linear term in Eq. (4) has been ignored. This model has been used only to study the influence of pure bending because of its simplicity for this case. The bending moment–axial force interaction is more involved, so it is not considered in this article for the exponential model.

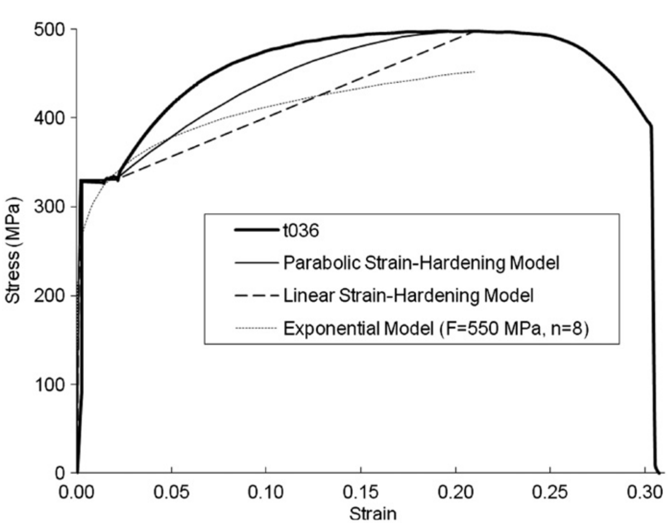

Fig. 1. Experimental stress–strain curve [2] and different models for mild steel.

The model parameters of the direct tensile test results for specimen t036 [2], whose stress–strain curve is shown in Fig. 1, have been used in the subsequent analysis. The strain-softening portion of the curve has been ignored in the present analysis.

3. Strain rate effect

The material strain rate effect for the dynamic yield and ultimate strength for a known value of strain rate, $\dot{\varepsilon}$, has been incorporated by using the Cowper–Symonds' equation:

$$\sigma_{yd} = \sigma_{ys}\left[1+\left(\frac{\dot{\varepsilon}}{A_y}\right)^{1/q_y}\right], \quad \sigma_{ud} = \sigma_{us}\left[1+\left(\frac{\dot{\varepsilon}}{A_u}\right)^{1/q_u}\right], \tag{5a, b}$$

where A_y, q_y and A_u, q_u are the Cowper–Symonds' coefficients for the yield and ultimate stresses, respectively. The values of these parameters are obtained from the results of uniaxial tensile tests for mild steel and are taken as: A_y=1300/s, q_y=5, A_u=6340/s, and q_u=5 which are reported in Ref. [19] for ε_y=0.05 and ε_u=0.25, respectively. The strain rate effect on the rupture strain has been ignored [2,20]. The compressive and tensile behaviour are assumed to be the same.

An estimate of the equivalent strain rate for a clamped beam struck by a mass at mid-span can be taken as [1]

$$\dot{\varepsilon} = \frac{V}{L}\sqrt{\frac{9H^2}{2L^2}+\frac{8k_s^2}{3}} \tag{6}$$