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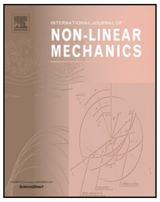

Influence of the Mullins effect on the stress–strain state of design at the example of calculation of deformation field in tyre

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1. Introduction

Mechanical properties of elastomers can be significantly improved by introduction of active filler nanosize particles. Carbon black was the first to be used for these purposes. At present, the mechanical properties of elastomeric nanocomposites continue to be intensively studied, as well as their dependence on the type of filler, its composition and production technique. For example, some of the latest investigations [1–7] study the effect of carbon black, carbon nanotubes, nanodiamonds, montmorillonite, palygorskite, schungite, etc. on elastomer properties. Along with experiments, other studies [8–15] perform mathematical modelling of elastomeric nanocomposite mechanical properties and the influence of structural features of the material on them.

An important feature of the materials under study is a softening in their mechanical properties as a result of preliminary deformation (the Mullins effect) [16–21].

To explain the material softening effect different hypotheses are expressed. A number of authors associate this phenomenon with the damage evolution in a rubber matrix. This can occur due to breakdowns in polymer chains, breakdowns of chemical cross-links between polymer chains, unravelling of existing entanglements and formation of cavities (vacuole). Other authors focus attention on the change in the material structure associated with the existence of filler particles. Breakdowns of aggregates and agglomerates of filler particles, decomposition of filler network can play an important role in the material softening. The third type of hypotheses is connected with the phenomena of interaction of the polymer with the filler. It is assumed that the polymer chains can slide over the surface of filler particles, tear away from them and reabsorb.

An example of modelling the softening effect of a hyperelastic material as a result of detachment of polymer chains from filler particles is presented in the article [22]. Breakdown of “polymer-filler” links and links between polymer chains is considered in the model of [23].

The authors of some works suggest a more comprehensive softening mechanism. Dargazany R. and Itskov M. observe the material as a system consisting of the filler network, the cross-linked polymer network and an additional polymer-filler network [24]. It is assumed that aggregates of filler particles can deform as a result of their decomposition. Fractal nature of filler aggregates and a possible change in the mutual arrangement of the particles are taken into account [25].

From the molecular positions, a description of the Mullins effect is given in the work [26]. Here, it is used the notion that the polymer network can be represented as a system of three networks: (1) a pure rubber network; (2) a network in which polymer chains irreversibly tear away from the surface of the particles; (3) a network in which polymer chains tear away and again adsorb with their ends on the surface of particles. The model allows to take into account hysteresis under repeated loading and anisotropy of the softening effect.

The constitutive equations, which can describe a large number of peculiarities of the softening effect, are given in [27]. It takes into account the possible breakdown of the overloaded polymer-filler regions during deformation and decrease in the elastomer inhomogeneity. Hysteresis is caused by continuous ongoing transformations in the material. There is a good agreement between theoretical results and experimental data at different deformation amplitudes and at different speeds. In the model there is 7 parameters, each of them has a clear physical meaning.

Along with theoretical studies, experimental works are continue to be performed, which allows obtain more detailed information on the Mullins effect. In the article [28] an analysis of the possible causes of the softening effect is presented and a conclusion that it is most likely a consequence of the adsorption and desorption of polymer chains on the filler surface is made. In a vacuo at the temperature of 80 and higher degrees the Mullins effect disappears for 3 days. A reestablishment of links between polymer chains and filler particles takes place.

Swelling in solvents also has a significant effect on the material behaviour both under compression [29] and under stretching [30]. In the swollen samples softening is less observed than in the dry ones. A solvent is able to reduce the interaction of polymer chains with fillers. Investigation of the growth mechanism of the materials strength during

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deformation allows conclude that softening occurs simultaneously with the destruction of the filler network [31,32].

Phenomenological modelling of the phenomenon is often performed within the framework of models of hyperelastic materials, which take into account the damage evolution with the help of scalar parameters [33,34]. In the work Mayau Li J., Lagarrigue D. the porosity of the medium (as an additional factor) is considered [35]. Comparisons of theoretical and experimental data are performed for uniaxial loading tests, equibiaxial loading tests and experiments with disc-shaped samples.

In order to develop the constitutive equations, which can describe the thermo-viscoelastic behaviour of a material with consideration of softening, the multiplicative decomposition of the deformation gradient into intermediate configurations and a scalar damping parameter are proposed in [36,37]. It is meant that the polymer network is a network of cross-linked molecular chains and networks of tangled and untangled molecular chains.

Another type is a model in which an internal variable is used to describe the viscoelastic behaviour, this variable is a tensor quantity [38]. In the work [39] it is suggested to take into account the deformation of the material, inelastic deformations (which include friction) and the variable of internal slippage in the model as three special summands of the energy potential.

An important feature of elastomers softening is the anisotropy of the material properties, it occurs after the first cycle of deformation. The study of the anisotropy peculiarities is carried out on samples, the second loading of which occurs at some angle with respect to the direction of the first loading [40,41].

Phenomenological models with internal variables but without specifying the physical phenomenon are proposed in the articles [42,43] and models based on the idea of the peculiarities of the interaction of polymer chains with filler particles are presented in [44,45].

In the present study we consider the Mullins effect in the different points of car tyre. An inhomogeneous softening of the material is observed in the process of using a rubber product. It turns out that at each of its points the mechanical properties must become different. This phenomenon is a consequence of inhomogeneous softening which depends on the maximal deformations at its points in the previous loading history. The influence of the softening effect on the stress–strain behaviour of the tyre under such conditions as acceleration and braking is of the greatest interest. Since an automobile tyre has its greatest load at acceleration and deceleration, there is a question how significant the softening will be and will it affect the operating ability of the tyre?

Focusing on the importance of taking into account the Mullins effect, we did not take into account the viscoelastic properties of the elastomer matrix. Our computational experiments show that even in a simplified formulation of the problem, there is a significant difference between calculations with and without taking the Mullins effect into account.

2. Modelling of mechanical properties of rubber

The automobile tyre has a complex structure. The main goal of this study is to find out how important it is to take into account the Mullins effect influencing the features of the stress–strain state of the object using computational experiments. Therefore, a simplified tyre structure is taken as an example (Fig. 1). It consists of only three materials. Real materials have viscoelastic properties. However, we must take into account the viscoelastic behaviour of rubber in the computational experiment. This is done in order to investigate the change in the strain maps that was caused by only one phenomenon — the softening of the material. Simultaneous consideration of two phenomena would be complicate the study. Taking viscoelasticity into account would make the analysis much more complicated, as in this case two dissipative mechanisms would work simultaneously at the first turn of the wheel rolling along the road. Therefore, the viscoelastic properties are not taken into account. All the irreversible phenomena within the framework of the study will be associated only with softening of rubber. The tyre sidewall and thread are made of material marked with 1 in the figure below. In the computational experiments, the Mullins effect is taken into account only in that material. Let us describe the mechanical properties of rubber in this part of the tyre using the modified Ogden–Roxburgh model [46,47].

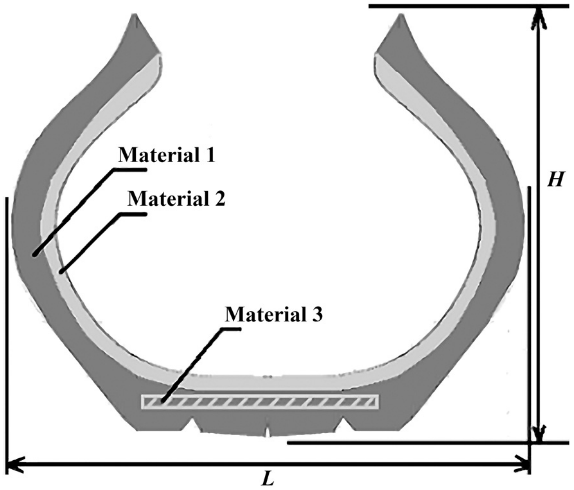

Fig. 1. The automobile tyre geometry the used to perform computational experiments.

According to this model , the mechanical properties are determined by the expression

$$U = \eta U_1 + U_2 + \phi(\eta). \tag{1}$$

in which

$$U_1 = \sum_{i=1}^{4} \frac{\mu_i}{\alpha_i}\left(\lambda_1^{\alpha_i} + \lambda_2^{\alpha_i} + \lambda_3^{\alpha_i} - 3\right) \text{ and } U_2 = \frac{1}{D}(J-1)^2,$$

where $\lambda_1, \lambda_2, \lambda_3$ are the extension ratio (eigenvalues of the left stretch tensor), μ_i, α_i are the constants determining elastic behaviour of the medium, D is the coefficient responsible for the volume change, $J = \lambda_1\lambda_2\lambda_3$ is the relative volume change, U_2– part of the energy that is spent on changing of the material volume. Since the material is incompressible, the constant D is taken close to zero on calculations performing. The special energy function $\phi(\eta)$ is calculated using the differential equation. This function may be interpreted as a measure of the energy required to cause the damage in the material.

$$\frac{\partial\phi}{\partial\eta} = -U_1, \tag{2}$$

where U_1 is represented as a function of η. The softening parameter $\eta \in (0;1]$ represents a dimensionless function, the value of which is calculated by the following formula:

$$\eta = \begin{cases} 1, & [U_1 = U_{\max}] \\ 1 - \frac{1}{r}\operatorname{erf}\left(\frac{U_{\max} - U_1}{m + \beta U_{\max}}\right), & [U_1 < U_{\max}] \end{cases} \tag{3}$$

where $U_{\max}$ is the maximum value of the potential U_1 at the previous load history, $\operatorname{erf}(x)$ is the error function. The value $\eta = 1$ corresponds to the condition without softening. Solving (3):

$$-U_1 = \left(m + \beta U_{\max}\right) \operatorname{erf}^{-1}\left(r\left(1-\eta\right)\right) - U_{\max}$$

and as a result, Equality (2) takes the form

$$\frac{\partial\phi(\eta)}{\partial\eta} = \left(m + \beta U_{\max}\right) \operatorname{erf}^{-1}\left(r\left(1-\eta\right)\right) - U_{\max}.$$

To determine the parameters of the binder, the sample was put to the uniaxial tension-compression experiment with pauses every time the

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