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Non-linear dynamics of size-dependent Euler–Bernoulli beams with topologically optimized microstructure and subjected to temperature field

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ABSTRACT

This paper is devoted to the investigation of non-linear dynamics of non-homogeneous beams with a material optimally distributed along the height and length of the beam. The study was initiated by topological optimization for the given boundary and loading conditions, which yielded maximum stiffness of a beam microstructure. As a result, the beam with an optimized microstructure exhibiting non-homogeneity in two directions, i.e. along beam thickness and length, was obtained.

In the second step, a beam model was derived based on the kinematic Euler–Bernoulli hypotheses and the modified couple stress theory including the von Kármán geometric non-linearity and heat flow action obeying the Duhamel–Neumann law.

Both static and dynamic behaviour of the optimized (non-homogeneous) and homogeneous beams were studied for different values of the material length-dependent parameter and temperature. Differences and peculiarities in static and dynamic problems were illustrated and discussed. In particular, the influence of the scale size parameter on chaotic beam dynamics was investigated. Also, scenarios of transition into deterministic chaos were detected and analysed for both homogeneous and optimized beams.

1. Introduction

Non-homogeneous materials (NM) are fabricated as multi-phase composites in which material properties change along the selected direction. This feature allows achieving better/required dynamic or static characteristics without a simultaneous occurrence of the concentration of stresses. Owing to this, NM beams are widely applied in gas and wind turbines, rotor blades of helicopters, bow feathers, and in numerous cosmic and marine structures [1].

The mentioned change in the material properties can be described either by pitch-like or exponential laws. For beams, the properties change can be realized either in the length or the thickness direction or in these two directions simultaneously.

Modification of elastic properties of the material, i.e. the microstructure properties of the structural members, have been widely used by practising engineers/designers.

The majority of scientific studies are focused on investigations of free NM beam vibrations, where the material change is introduced along the beam thickness. The main target of the studies is to determine the beam frequencies with respect to different forms of beam vibration. A few different methods and methodologies have been employed for this purpose [2–8]. It has been observed that the problems of free vibrations of beams with non-homogeneous transverse cross sections or with material properties changing in the longitudinal direction exhibit more complexity comparing with the classical investigations of beams with homogeneous transverse cross sections and made of homogeneous materials [9,10] due to the occurrence of time-dependent coefficients in the governing differential equations. This is why the majority of the carried out investigations rely on numerical methods [11,12]. For instance, non-linear behaviour of functionally graded beams with the von Kármán geometric non-linearity has been studied in reference [13] by using FEM (finite element method). Kien [14] has considered large displacement response of the tapered cantilever beams made of axially functionally graded material. Alshorbagy et al. [15] have studied free vibration characteristics of a functionally graded beam also by using FEM. Shahba et al. [16–18] have carried out investigations of free vibration and stability of axially functionally graded tapered Euler–Bernoulli and Timoshenko beams with classical/non-classical boundary conditions by using a few numerical techniques. An analytical study of

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non-linear vibrations of functionally graded beams has been carried out by Ke et al. [19] with the use of the first-order Galerkin method.

In order to analyse vibrations of non-uniform and exponentially functionally graded and tapered beams, numerous mathematical models based on the Euler–Bernoulli [9,20–26] and Timoshenko [17,27–30] hypotheses have been employed. Authors of the Refs. [31–33] have studied free vibrations of clamped tapered beams on linear elastic foundations, axially functionally graded tapered Euler–Bernoulli beams, and axially functionally graded Timoshenko beams having non-uniform cross sections.

It should be noted that the classical mechanics of rigid bodies does not allow one to interpret or forecast the size-dependent behaviour of the MEMS/NEMS structural members due to a lack of a size-dependent parameter responsible for quantifying micron and sub-micron scale processes playing a crucial role in the final global dynamic behaviour of the mentioned structural elements. So far, the following theories have been used for modelling of the scale effects occurred in a continuum: the couple-stress theory of elasticity [34,35], the non-local theory of elasticity [36], the strain gradient theory of elasticity [37], and the general theory of curved deformable interfaces in solids [38].

A theoretical foundation for the couple stress-based strain gradient theory for elasticity has been proposed by Yang et al. [39]. In that paper, the governing equations contain, in spite of two classical Lame constants, an additional higher-order material constant. This theory has been employed and validated by numerous researchers to get a reliable interpretation of the size-dependent dynamic behaviour of microstructures [40–43].

Rajabi and Ramezani [44] have derived PDEs for a geometrically non-linear homogeneous Euler–Bernoulli beam by using the von Kármán relations. The Galerkin method in the first approximation has been employed and an influence of a size-dependent coefficient of the magnitude equal to the fundamental frequency of non-linear vibrations has been studied.

The nonlinear forced vibrations of a microbeam have been investigated in [45], employing the strain gradient elasticity theory. The geometrically nonlinear equation of motion, the microbeam, taking into account the size effect, has been obtained employing a variational approach. Results of the frequency response of the system have been studied.

Arbind and Reddy [46] have considered functionally graded microstructure-dependent beams made from a material non-homogeneously distributed along the beam thickness, taking into account the von Kármán non-linearity. The modified couple stress theory has been used and the counterpart governing equations for the Euler–Bernoulli and the Timoshenko beams have been derived. In particular, the influence of the scale length parameter, the law of the beam properties change along the thickness, shear deformations, and geometric non-linearity on the static beam deflections have been studied. Arbind et al. [47] have derived non-linear PDEs of the size-dependent Sheremetev–Pelekh–Reddy beam made from NM by using Hamilton's principle.

In the present work, we are aimed at carrying out a comparative analysis of the static and dynamics behaviour of size-dependent beams [48,49]. An investigation of non-linear dynamics of micro/nanobeams, plates, and shells has been carried out in Refs. [50–52]. The studies, which can be found in the available literature devoted to the microstructure of beams, have been conducted without using the optimization methods. Furthermore, the overviewed references did not offer a rigorous analysis of the reliability of results. Also, there is a lack of works devoted to studying the chaotic dynamics of size-dependent beams made from NM in the existing literature, which stands for the motivation of the present study. In this work, a mathematical model of a functionally graded size-dependent non-linear Euler–Bernoulli beam is derived, the algorithms and programmes for construction of a microstructure beam (working in the given conditions), and then investigation of its static/dynamic features are developed.

At the first stage of our analysis, for the given conditions of the beam loading, boundary conditions, and the temperature field, the topological optimization was introduced following the criterion of keeping the maximum stiffness. In all case studies, the achieved optimal microstructure is original and not obtained by others. As a result of the carried out optimization, the optimal microstructure beam exhibits functional grading in two directions, i.e. along its length and thickness.

The second stage of our study was aimed at analysing static and dynamic beam behaviour. The mathematical model is constructed based on the Euler–Bernoulli hypotheses, the modified couple stress theory, and the von Kármán geometric non-linearity. The influence of the temperature field on the model follows the classical Duhamel–Neumann relations.

The carried out comparative analysis of the statics and non-linear dynamics of homogeneous and non-homogeneous (functionally and optimally graded) beams yielded the main observation that the stress–strain states and magnitudes of the fundamental frequencies of vibrations of these structures essentially depend on the used temperature.

The paper is organized in the following way. The mathematical background with emphasis put one the topological optimization and the theory of the size-dependent Bernoulli–Euler beam is given in Section 2. Section 3 presents the method used to find the numerical solution, static and dynamic problems of the size-dependent homogeneous, and non-homogeneous Euler–Bernoulli beams subjected to a temperature field, and the detected scenarios of transition of the beam vibrations from the regular to chaotic dynamics. Section 4 contains conclusions for the carried out study.

2. Mathematical background

2.1. Topological optimization

Consider a 2D elastic beam area Ω, bounded by the closed surface $\Gamma = \Gamma_1 \cup \Gamma_2 \cup \Gamma_3$ and subjected to a plane stress state. We assume that the material of beams is linearly elastic and isotropic and that it is subjected to the temperature field $T(\mathbf{x}), \mathbf{x} = \{x_1, x_3\}$.

The computational model with the scheme of boundary conditions is reported in Fig. 1, and $\theta = T(\mathbf{x}) - T_0$ stands for the temperature change with respect to the initial temperature T_0. The boundary Γ_1 corresponds to the clamping–clamping boundary conditions, whereas Γ_2 stands for the free surface part, i.e. without loading. Boundary surface Γ_3 is under action of vertical load $t_2 = q$ acting in the direction of the axis OX_3.

In the case of the displacements field (u_1, u_3), the governing equilibrium equation takes the following form

$$\sigma_{ij,j} = 0 \text{ in } \Omega, \tag{1}$$

where σ_{ij} stands for the stress tensor. The relationship between linear deformations and displacements obeys the following relation

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \; i, j = 1, 2. \tag{2}$$

The corresponding stress–strain relationship follows the Duhamel–Neumann law [53]

$$\sigma_{ij} = E(\mathbf{x})(\varepsilon_{ij} - \alpha\,\theta\,\delta_{ij}) \tag{3}$$

where $E(\mathbf{x})$, $\alpha(\mathbf{x})$, $\theta(\mathbf{x})$ and δ_{ij} denote Young's modulus, the temperature coefficient of linear extension of the non-homogeneous material of the beam Ω, the difference between the current and initial beam temperatures, and the Kronecker symbol, respectively. Recall that the displacement and temperature fields are coupled by Eq. (3).

With the help of the topological optimization, the optimal beam material distribution can be found, which allows one to obtain either maximum stiffness or minimum flexibility of the beam thermoelastic body.