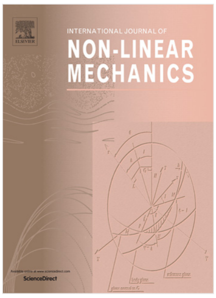

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On the Newton-Raphson basins of convergence of the out-of-plane equilibrium points in the Copenhagen problem with oblate primaries

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Abstract

The Copenhagen case of the circular restricted three-body problem with oblate primary bodies is numerically investigated by exploring the Newton-Raphson basins of convergence, related to the out-of-plane equilibrium points. The evolution of the position of the libration points is determined, as a function of the value of the oblateness coefficient. The attracting regions, on several types of two-dimensional planes, are revealed by using the multivariate Newton-Raphson iterative method. We perform a systematic and thorough investigation in an attempt to understand how the oblateness coefficient affects the geometry of the basins of convergence. The convergence regions are also related with the required number of iterations and also with the corresponding probability distributions. The degree of the fractality is also determined by calculating the fractal dimension and the basin entropy of the convergence planes.

Keywords: Circular restricted three-body problem, Oblateness coefficient, Basins of convergence, Fractal basin boundaries

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1. Introduction

The classical circular restricted three-body problem still remains, without any doubt, one of the most intriguing and open topics in celestial mechanics and dynamical astronomy. According to [38] the restricted three-body problem describes the motion of a third body, with an infinitesimal mass (thus acting as a test particle), inside the combined gravitational field of two primary bodies. This topic has numerous practical applications which expand from molecular physics, to chaos theory, planetary physics, as well as to galactic dynamics.

Over the last decades, the classical three-body problem has been substantially modified in an attempt to describe more realistically the nature of motion of massless test particles in the Solar System, by taking into consideration additional dynamical parameters of the system. In particular, the effective potential of the classical restricted three-body problem has been upgraded by including several types of additional forces.

The two primaries are spherical and homogeneous in the classical version of the restricted three-body problem. However, several celestial bodies in our Solar System (e.g., Saturn and Jupiter) have in fact an oblate shape [6]. In order to obtain a much more realistic description of the motion of the test particle in the vicinity of such oblate bodies the parameter of the oblateness has been introduced. The influence of the oblateness on the character of motion has been investigated in a series of papers (e.g., [2, 7, 13–15, 18, 19, 23, 27–34, 40, 41]).

Another issue of great importance in dynamical systems is the so-called "basins of convergence" associated to the equilibrium points. These convergence regions reveal how each point on a two-dimensional plane is attracted by the equilibrium points of the system, when an iterative method is used for numerically solving the system of the first order derivatives of the effective potential function. In the literature there is a plethora of numerical methods for numerically solving an equation with only one variable. For a system of equations, with two or more variables, on the other hand only a couple of methods exist. The most famous one is the classical Newton-Raphson method, while there is also the Broyden's method [8], which however is in fact a quasi-Newton method. In numerous previous studies the Newton-Raphson iterative scheme has been used for determining the corresponding basins of convergence in several types of Hamiltonian systems (e.g., the Hill problem with oblateness and radiation pressure [11, 43], the restricted three-body problem, where the primaries are magnetic dipoles [16], the restricted three-body problem with oblateness and radiation pressure [42], the restricted four-body problem [5, 17, 35, 36], the restricted five-body problem [45], the ring problem of $N + 1$ bodies [9], or even the pseudo-Newtonian restricted three-body problem [44]).

In dynamical system knowing the exact positions of the equilibrium points is an issue of paramount importance. Unfortunately, in many systems, such as those of the N-body problem (with $N \geq 3$), there are no explicit formulae

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