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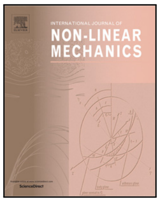

Modeling and mechanical analysis of multiscale fiber-reinforced graphene composites: Nonlinear bending, thermal post-buckling and large amplitude vibration

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ABSTRACT

In this paper, a mathematical model was developed to predict the effective material properties of graphene nanoplatelets/fiber/polymer multiscale composites (GFPMC). The large deflection, post-buckling and free nonlinear vibration of graphene nanoplatelets-reinforced multiscale composite beams were studied through a theoretical study. The governing equations of laminated nanocomposite beams were derived from the Euler–Bernoulli beam theory with von Kármán geometric nonlinearity. Halpin–Tsai equations and fiber micromechanics were used in hierarchy to predict the bulk material properties of the multiscale composite. Graphene nanoplatelets (GNPs) were assumed to be uniformly distributed and randomly oriented through the epoxy resin matrix. A semi-analytical approach was used to calculate the large static deflection and critical buckling temperature of multiscale multifunctional nanocomposite beams. A perturbation scheme was also employed to determine the nonlinear dynamic response and the nonlinear natural frequencies of the beams with clamped–clamped, and hinged–hinged boundary conditions. The effects of weight percentage of graphene nanoplatelets, volume fraction of fibers, and boundary conditions on the static deflection, thermal buckling and post-buckling and linear and nonlinear natural frequencies of the GFPMC beams were investigated in detail. The numerical results showed that the central deflection and natural frequency were significantly improved by a small percentage of GNPs. However, addition of GNPs led to a lower critical buckling temperature.

1. Introduction

GNPs/fiber/polymer multiscale laminated composite materials consist of high strength and modulus fibers, graphene nanoplatelets (GNPs) and a polymer matrix. GNPs/fiber/polymer multiscale composites can significantly reduce weight with the same or better performance than that of metal materials and, as such, have drawn significant attention in recent years. GNPs/fiber/polymer composites have been widely used in many structural applications, such as aircrafts, space vehicles, automotive industry, sporting goods, and marine industry in which weight reduction is crucial for higher speeds and increased payloads. GNPs are excellent candidates for nanoscale reinforcement toward improvement of multifunctionality of fiber-reinforced composites. They are expected to have extraordinary properties such as high thermal conductivity, superior mechanical properties, and superb electronic transport properties. These essential properties of graphene have generated huge attention for their possible application in several devices [1]. Graphene may be preferred over other conventional nanofillers (such as sodium montmorillonite, carbon nanotube (CNT), carbon nanofiber (CNF), graphite, and exfoliated graphite (EG)), due to high surface area, aspect ratio, tensile strength, thermal and electrical conductivity, electromagnetic interference (EMI) shielding ability, flexibility, transparency, and low coefficient of thermal expansion [2].

The analytical Halpin–Tsai model can predict the material properties of composite materials based on the geometry and orientation of the filler and the elastic properties of the filler and matrix [3]. An excellent survey of the research on different modeling techniques for predicting the mechanical behavior of polymer composites was presented by Valavala and Odegard [4]. Thostenson and Chou [5] showed that MWNT/polystyrene composite elastic properties are sensitive to nanotube diameter by an approach based on the Halpin–Tsai micromechanical method. Mechanical properties of high-density polyethylene composites reinforced with CNTs were presented by Kanagaraj et al. [6]. They employed both the Halpin–Tsai model and a modified form of

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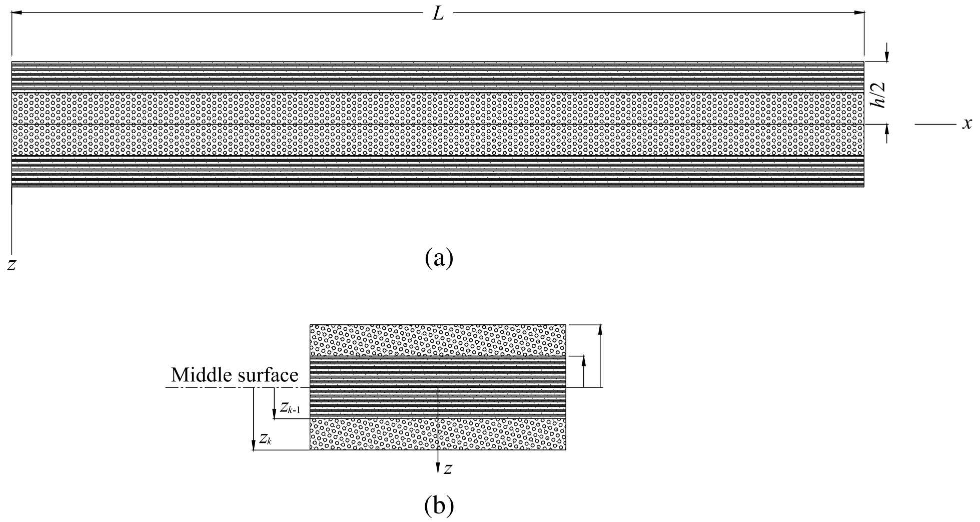

Fig. 1. Configuration of a multilayered GNPs/fiber/epoxy multiscale laminated composite beam: (a) front view and (b) cross section.

the rule of mixture model to make a comparison between theoretical and experimental results. Rafiee [3] used the Halpin–Tsai equations and fiber micromechanics in hierarchy to predict the bulk material properties of the multiscale composite. This approach was used further to study the vibration of nanocomposite beams [7]. Recently, Rafiee et al. [8] addressed the cross-sectional design and analysis of fiber-reinforced multiscale composite beams of general cross-sectional shape and arbitrary anisotropic material properties using Halpin–Tsai equations and fiber micromechanics. The influence of CNTs weight percentage and volume fraction of fibers was investigated in their work.

Most of the previous research regarding graphene-reinforced composite materials has mainly focused on the characterization and modeling of two-phase composites. Reports on the experimental development of advanced polymer composites reinforced by graphene and reduced graphene oxide have been given in the literature [9–19]. There few works devoted to the morphological and mechanical properties of multiscale multifunctional GFPMCs [20–24]. For instance, Kamar et al. [20] investigated the ability of GNPs to improve the interlaminar mechanical properties of glass-reinforced multilayer composites. Their results showed that the impact-side damage area decreased with increasing concentration of GNPs, while the back-side damage area increased. Hsieh et al. [21] studied the effect of environmental aging on interlaminar properties of graphene nanoplatelets reinforced epoxy/carbon fiber composite laminates. Their experimental results showed that the composite laminates containing graphene nanoplatelets presented appreciable improvement. Recently, Seretis et al. [24] investigated the bending and tensile performance of hand lay-up produced glass fabric/epoxy laminated composites after matrix reinforcement with pre-dried graphene nanoplatelets (GNPs) up to 30% wt. content.

Structural modeling of CNTs and GNPs-reinforced laminated composite beams has been a topic of active research in the recent years. Among those available in the literature, Rafiee and his coworkers developed a structural model for the analysis of three-phase multiscale laminated CNTs-reinforced composite beams ([7,25,26]) and plates ([3,27,28]). For beam-type structures, they studied the bending [26] and free vibration [7,26] of carbon nanotubes-reinforced multiscale laminated composite beams. Static bending [3,27], and free vibration [27,28] were also investigated for carbon nanotubes-reinforced laminated multiscale composite plates. Very few studies related to the structural modeling of two-phase GNPs-reinforced composite beams are available in the literature [29–31]. Modeling and analysis of thermal postbuckling and nonlinear bending of graphene-reinforced composite (GRC) laminated beams in a thermal environment and resting on elastic foundations have been presented by Shen et al. [29]. Yang et al. [30] studied the buckling and postbuckling of functionally graded multilayer GNPs-reinforced laminated composite beams resting on an elastic foundation from the perspective of a first-order shear deformation theory. Feng et al. [31] investigated the linear and nonlinear free vibration behaviors of nanocomposite beams reinforced with non-uniformly distributed GNPs within the framework of Timoshenko beam theory and von Kármán nonlinear strain–displacement relationship. They found that adding a very small amount of GPL to the polymer matrix can significantly increase the natural frequency of the composite beam.

Despite the great importance of graphene multifunctional composites, no mathematical model has been available for the simulation of bending, buckling and vibration response of graphene nanoplatelets/fiber/polymer laminated multiscale composite beams.

To fill this gap, the nonlinear static, thermal bifurcation buckling and postbuckling, and free vibration of GNPs/fiber/polymer laminated composite beams were investigated in the present work. The governing equations of the GFPMC multiscale laminated beam were derived from the Euler–Bernoulli beam theory and the von Kármán geometric nonlinearity. Halpin–Tsai equations and fiber micromechanics were used to predict the bulk material properties of the multiscale composite. The GNPs were assumed to be uniformly distributed and randomly oriented through the epoxy resin matrix. The Galerkin procedure yielded a time-wise equation with cubic nonlinear terms. The method of multiple time scales was employed to determine the dynamic response and nonlinear natural frequencies of the beams with clamped–clamped, and hinged–hinged boundary conditions. The effects of weight percentage of GNPs, boundary conditions, and volume fraction of fibers on large deflections, critical buckling temperature and linear and nonlinear frequencies of the GFPMC beams were investigated through a comprehensive parametric study.

2. Governing equations

2.1. Material properties

A GFPMC beam of length L and thickness h is shown in Fig. 1. The three-phase GFPMC multiscale beam was assumed to be made of a mixture of isotropic matrix (epoxy resin), GNPs and fibers (E-Glass) with different fiber alignment for each lamina through the thickness. The graphene nanoplatelets composite was regarded as isotropic, as the GNPs were assumed to be uniformly distributed and randomly oriented through the matrix. It was also assumed that the GNPs-matrix bonding and GNPs dispersion in the matrix were perfect, that each GNP had the same mechanical properties and aspect ratio, that all GNPs were