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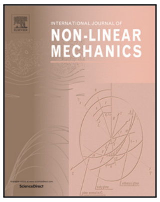

Numerical nonlinear frequency analysis of pre-damaged curved layered composite structure using higher-order finite element method

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ABSTRACT

The influence of the internal debonding on the structural stiffness and the nonlinear modal characteristics of the layered structure are examined extensively in the current research article. For the investigation purpose, the shell frequency responses are obtained numerically for both the linear and the nonlinear cases via a generic type of mathematical formulation using the Equivalent Single Layer (ESL) theory in the framework of two kinematic models. The current formulation not only includes the influence of the transverse shear deformations but also satisfies the parabolic variation of transverse shear stress through the thickness. Additionally, the geometrical nonlinear distortion modeled via Green–Lagrange strain–displacement relations. Further, the internal debonding between the adjacent layers are modeled using sub-laminate approach and the displacement continuity between segments (laminate and delaminate) have been established through the intermittent continuity conditions. The nonlinear system governing equation of the vibrated structure is obtained via Hamilton's principle and converted to set of nonlinear algebraic equations through the isoparametric finite element (FE) steps. The desired responses are solved numerically with the help of robust (direct iterative method) technique and compared with available results to demonstrate the solution accuracy. Subsequently, an adequate number of examples are solved for the delaminated structure using the current higher-order nonlinear models and the influential parameters discussed in detail.

1. Introduction

Since last few decades, many researchers across the globe have put their special attention to the development of new composite materials and subsequent application in the high-performance engineering field. Owing to the knowledge in the domain of fabrication of composite structure not only help to design of structural component but also need the data related to the strength, stiffness and their final performances under the influence of static and/or dynamic loading. This, in turn, allows the design engineer to explore the application of the layered composite structural materials in the new dimension. Nowadays all most all the weight sensitive industries such as aerospace, marine, aircraft, modern civil construction etc. are treating the composite materials as a potential candidate and most suited material for the structural application. Though the composite materials have found many applications, the application in aerospace and aeronautical industries are more challenging due to the complicated geometries (curved), loading condition (internal pressure, fluid–structure interaction) and material with less-known material properties. In view of the above, the numerical techniques including the new mathematical approaches are adopted every now and then for the accurate prediction of the structural performance exposed to the large deflection (due to the flexibility) under the influence of large amplitude excitations.

In this regard, the major research findings reported in open literature either focus on the mathematical modeling of mid-plane kinematics (accurate evaluation of deformation behavior) or the nonlinear strain–displacement including the solution techniques (analytical, numerical and exact). It is important to mention that the most of the research article discussed on von-Karman type of nonlinear strain for the modeling of the large geometrical distortion and it constitutes by considering the moderate rotations only. While the total structural deformation depends on three distinct parameters (translation, rotation and distortion) instead of any one of them. However, the combined influence of the total deformation including all of the nonlinear higher-order strain terms on the frequency behavior of the delaminated structure is not studied extensively. Now, the necessity and importance of the current research have been established by discussing few relevant research articles in the following lines.

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The nonlinear frequency responses of the hybrid composite plate structure are solved using a new analytical technique by Lee and Kim [1]. Further, the layered structural responses including the different size of debonding have been obtained via the higher-order shear deformation theory (HOSDT) in the past by different researcher (Reddy [2], Huang and Zheng [3] and Parhi and Singh [4,5]) to establish the importance of the higher-order kinematics. Additionally, various research article also reported (Shin [6] and Naidu and Sinha [7]) the kinematic modeling of the layered composite using the first-order shear deformation theory (FOSDT) including the new approximation technique (Galerkin's approximate in association with Runge–Kutta method) by Chien and Chen [8]. An inclusive review on the theories and the subsequent analysis of nonlinear vibration characteristic of the doubly curved shell structure reported by Alijani and Amabili [9]. In continuation to that the implementation of differential quadrature method (DQM) for the computation of nonlinear frequency responses of the smart composite structure by Kolahchi and his co-authors [10,11]. Similarly, the dynamic behavior of the curved panel composite structure investigated by Tornabene and his co-researcher using various numerical techniques, i.e., the strong form of finite element [12], the weak formulation steps for the isogeometric analysis [13] and layer-wise approach [14].

Though an ample number of studies related to the mathematical formulation of the layered structure has been developed and subsequently implemented, however, the justification interrelated for the comprehensive behavior may need more clear understanding. It is because of the fact that the certain unavoidable phenomena such as delamination may arise either during manufacturing or because of the operational conditions and this, in turn, reduces the structural stiffness which impacts the absolute structural performances largely. The prediction of structural responses of the laminated structure without considering the influence of the debonding limits the knowledge about the actual structural strength and integrity as well as the life expectancy. Therefore the consideration of the effect of debonding is highly important while predicting the structural responses whether under the influence of the static and/or dynamic loading. Consequent upon few relevant literature related to the delaminated structure is discussed in the following line for the concurrence of the present research objective.

Initially, the dynamic behavior of the delaminated structure is investigated with the help new higher-order zig-zag theory by Cho and Kim [15] and the model extended further to examine the frequency responses of the multi delaminated layered structure by Oh et al. [16]. In addition, the influence of the damage (transverse crack) on elastic property degradation and stiffness reduction of the simple laminated and the hybrid composite structure are studied extensively by Tounsi and his research group [17–19]. Further to add the knowledge regarding solution techniques (numerical and analytical) of the debonded layered structure a comprehensive review article published by Della and Shu [20] mainly discussed on the evaluation of modal responses. Likewise, the dynamic characteristics of the internally damaged composite and sandwich structures (beam and plate) are studied using the HOSDT kinematic model in association with finite element method (FEM) by Hu et al. [21] and Schwarts-Givli et al. [22], respectively. Further, the effect of geometrical nonlinearity on the debonded sandwich structural responses is investigated in the framework of the HOSDT mid-plane kinematics by Frostig and Thomsen [23]. Also, new mathematical models are proposed and developed using the well-established kinematic theories (FOSDT [24,25] and HOSDT [26,27]) for the debonded orthotropic plate structure by Szekrenyes including the continuity and exact kinematic conditions. In continuation to the earlier studies, the free vibration frequency characteristics of the debonded beam structure are computed using the FE technique in the framework of Euler–Bernoulli beam theory by Szekrenyes [28]. The modal responses of the damaged layered structure computed numerically using the FE technique in association with FOSDT kinematics under the hostile environment and verified with their own experimental data by Panda et al. [29]. Later, the fundamental frequency responses of the debonded graphite/epoxy layered composite conical shell structures are computed via the versatile FE technique using Mindlin's plate theory by Dey and Karmakar [30,31] and Bandyopadhyay et al. [32]. In addition, the nonlinear dynamic characteristics of the damaged layered composite and sandwich structures are investigated by Marjanovic et al. [33] using the FE technique including the geometrical distortion via von-Karman nonlinear strain kinematics.

The present review clearly indicates that the linear/nonlinear frequency analysis of shell structure has received limited attention in comparison to the beam and/or flat debonded structural components. Similarly, the effect of internal debonding on the final structural responses and the structural integrity has not studied extensively with reference to the edge debonding. Lastly, the major research related to the nonlinear frequency analysis of intact or damaged structure mainly utilized von-Karman type of strain kinematics which include a part of the total structural deformation. Hence, to bridge the gap between the past studies, the current research aims to develop first time a generic type of mathematical model for the investigation of the nonlinear frequency parameter of the internal pre-damaged layered composite curved panel structure. In this regard, the excess geometrical distortion including the internal debonding has been modeled via Green–Lagrange strain kinematics and sub-laminate approach in the framework two higher-order kinematic theories. Further, the applicability and the adaptability of the current numerical solutions are verified via concurrent convergence and comparison test. Finally, an adequate number of numerical examples are solved for the internally damaged shell configurations to show the influence of various parameter associated with geometry and material properties including the role of delamination (size and position) on the nonlinear frequency responses.

2. Geometrical configuration and mathematical explanation of panel structure

For the numerical analysis purpose, the layered curved shell structure (consist of 'N' number of uniformly thick orthotropic layers as in Fig. 1) is modeled mathematically using the available geometrical parameters (length 'a', width 'b' and thickness 'h') along the corresponding coordinate axes (ξ_1, ξ_2 and ζ-direction). In addition, the principal radii of curvature i.e., R_{ξ_1} and R_{ξ_2} (along the ξ_1 and ξ_2-directions) at the mid-plane ($\zeta = 0$) are included in the mathematical model to achieve the desired curved geometries by setting the twist radius of curvature as infinite ($R_{\xi_1\xi_2} = \infty$). The global coordinate system and the corresponding reference frame i.e., ξ_1, ξ_2 and ζ is defined at the mid-plane of the panel (refer Fig. 1). Now, the different shell configurations (single and doubly curved panel) are achieved by choosing the suitable curvature sequence say, cylindrical shell $\left(R_{\xi_1} = R,\ R_{\xi_2} = \infty\right)$, spherical $\left(R_{\xi_1} = R,\ R_{\xi_2} = R\right)$, elliptical $\left(R_{\xi_1} = R,\ R_{\xi_2} = 2R\right)$, hyperboloid $\left(R_{\xi_1} = R,\ R_{\xi_2} = -R\right)$ and plate $\left(R_{\xi_1} = \infty,\ R_{\xi_2} = \infty\right)$.

2.1. Generalized displacement field kinematics

In this section, the detailed regarding the mathematical modeling of layered composite continuum panel structure is discussed. For the modeling purpose, two higher-order kinematic theories are employed namely, HOSDT-1 and HOSDT-2. The models are expressed via nine (Reddy and Liu [34]) and ten (Kishore et al. [35]) space variables to derive the exact deformation kinematics and the conceded in the following form:

HOSDT.-1:

$$\left.\begin{aligned}\bar{u}\left(\xi_1,\xi_2,\zeta\right) &= u_0 + \zeta\phi_1 + \zeta^2\psi_1 + \zeta^3\theta_1\\ \bar{v}\left(\xi_1,\xi_2,\zeta\right) &= v_0 + \zeta\phi_2 + \zeta^2\psi_2 + \zeta^3\theta_2\\ \bar{w}\left(\xi_1,\xi_2,\zeta\right) &= w_0\end{aligned}\right\} \tag{1}$$