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The Rayleigh–van der Pol oscillator on linear multibody systems

René Bartkowiak *, Christoph Woernle

University of Rostock, Chair of Technical Mechanics/Dynamics, 18051 Rostock, Germany

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ABSTRACT

In nature many interactions of different oscillatory systems can be described by the phenomenon of synchronization. This phenomenon can be widely used for engineering applications. In this contribution, the synchronization between a self-sustained oscillator and linear multibody systems with harmonic force excitation is used successfully to damp unwanted vibrations of the host structure and additionally to harvest energy from the load. To this end, the parameters of the Rayleigh–van der Pol oscillator are adjusted to a given harmonic base excitation in such a way that the response is harmonic as well. Thereby the amplitude of the response and the phase-shift between the excitation and the oscillator response remain free. Afterwards the oscillator is attached to a forced multibody system and the amplitude and phase-shift are adjusted for the tasks of vibration absorption and energy harvesting.

1. Introduction

Classical tuned mass dampers (TMD) have been successfully used for the suppression of unwanted vibrations of dynamically loaded structures [1]. The drawback of passive TMDs is that they become detuned when the parameters of the host structure change. To avoid such detuning, active TMDs with adaptive stiffness and damping ratio were developed. A disadvantage of active TMDs is the necessity of an external power supply. Therefore, newer approaches to active TMDs harvest and accumulate energy from the loaded structure, enabling self-sufficient operation [2,3].

In the most approaches, the parameters of the mass damper are always tuned for desired stationary solutions of the whole system. To improve the transient behavior, the TMDs can be replaced by actively controlled self-sustained oscillators with the same stationary behavior. Then the oscillator can synchronize with the system motion, and the stationary solution of the whole system can be reached much faster due to the nonlinear damping behavior of the oscillator.

Self-sustained oscillators are autonomous dynamical systems with a stable limit cycle in phase space. Because of this, oscillators have stable periodic motions with a fundamental angular frequency ω_o, called natural or partial angular frequency, and a free phase angle, depending on the initial conditions. If the oscillator is forced by a harmonic force with the angular excitation frequency Ω, which can be regarded as the most simple case of synchronization [4], then the fundamental angular frequency of the driven oscillator ω_{dr} can become a multiple of the driving angular frequency Ω, thus $\omega_{dr} = \frac{k}{p}\Omega$, with the whole numbers k, p called k/p-synchronization [5]. Here only $1/1$-synchronization is considered. If $1/1$-synchronization exists and is stable, then a constant phase-shift $\Delta\alpha$ exists between the first harmonic of the oscillator motion and the excitation, also called phase-locking. If the natural angular frequency of the autonomous oscillator equals the angular excitation frequency, thus $\omega_o = \Omega$, then the synchronized oscillator motion can be in-phase or in anti-phase to the excitation, thus $\Delta\alpha = 0$ or $\Delta\alpha = \pi$, depending on the parameters of the oscillator and the strength of excitation. In both cases the energy transfer between the excitation and the oscillator, averaged over the period $\frac{2\pi}{\Omega}$, is zero. In case of differing frequencies, $\omega_o \neq \Omega$, also called detuning, a positive or negative angle of lag from the in-phase or anti-phase motion occurs, and energy is transferred between the excitation and the oscillator.

In [6] it was shown that the parameters of the Rayleigh–van der Pol (**RvdP**) oscillator [7] can be adapted to a given harmonic excitation in such a way that the oscillator steady state response is harmonic as well, whereby the amplitude and the constant phase shift between the excitation and the response of the oscillator can be arbitrarily prescribed. If this oscillator is now attached to a mass of a linear multibody system, the motion of this coupling mass can be seen as a base excitation of the oscillator. Exciting the multibody system by an external excitation force yields to a harmonic vibration behavior of the coupling mass and by this to a harmonic response of the RvdP oscillator. Since the amplitude and the phase shift of the oscillator response can be freely defined by its parameters, the interaction of the oscillator response and

* Corresponding author.
E-mail address: rene.bartkowiak@uni-rostock.de (R. Bartkowiak).

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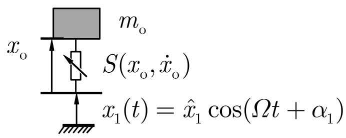

Fig. 1. RvdP oscillator with base excitation $x_1(t)$.

the multibody system can be used for active vibration absorption of the excited multibody system as well as for energy harvesting.

In this contribution, the synchronization between the nonlinear RvdP oscillator and the motion of a harmonically forced linear multibody system is studied. To this end, the parameters of the RvdP oscillator are adjusted for the tasks of active vibration absorption as well as for energy harvesting.

The paper is organized as follows: In Section 2 the parameters of the RvdP oscillator are derived for a given harmonic base excitation in such a manner that the synchronized steady-state oscillation is harmonic as well. Thereby the amplitude and phase angle of the oscillator motion remain free parameters. In Section 3, the obtained oscillator is attached to a mass–spring–damper system with harmonic force excitation. After solving the existence conditions for 1/1-synchronization, the stability area of the synchronization is studied. Then the amplitude and phase angle of the oscillator motion are adjusted for the vibration absorption task and the energy harvesting task, respectively, taking the existence and stability condition of synchronization into account. Section 4 generalizes these results for an RvdP oscillator attached to a general linear multibody system with harmonic force excitation. Although the topology of the multibody system is not specified, the existence conditions for 1/1-synchronization and predictions for the stability during vibration absorption mode and energy harvesting mode can be determined. Results are visualized by numerical examples.

2. The driven Rayleigh–van der Pol oscillator

Let us consider the system in Fig. 1 comprising a mass m_o described by coordinate x_o, a state-dependent force element $S(x_o, \dot{x}_o)$ and a harmonic base excitation $x_1(t) = \hat{x}_1 \cos(\Omega t + \alpha_1)$ with the constant amplitude $\hat{x}_1$, the constant angular excitation frequency Ω and the phase angle α_1.

By using the speed gradient method [8] a state-dependent control law $S(x_o, \dot{x}_o)$ can be obtained that guarantees asymptotically stable periodic motions $x_o(t)$ or limit cycles of the unforced system, thus $\hat{x}_1 = 0$. Here, the force element is composed by a virtual linear spring element with stiffness coefficient c_o and a virtual nonlinear damping element,

$$S(x_o, \dot{x}_o) = S_{\mathrm{damp}}(x_o, \dot{x}_o) - c_o x_o. \tag{1}$$

The equation of motion for the autonomous system according to Fig. 1 with $\hat{x}_1 = 0$ then reads

$$m_o \ddot{x}_o + c_o x_o = S_{\mathrm{damp}}(x_o, \dot{x}_o). \tag{2}$$

In order to obtain asymptotically stable periodic motions, a control-Lyapunov function is defined by

$$Q = \frac{1}{2}(H(x_o, \dot{x}_o) - H_o)^2 \tag{3}$$

with the total energy of the system $H(x_o, \dot{x}_o) = \frac{1}{2} m_o \dot{x}_o^2 + \frac{1}{2} c_o x_o^2$ and the desired constant energy $H_o > 0$. Fulfilling the requirement $\dot{Q} < 0$ the control scheme

$$\begin{aligned} S_{\mathrm{damp}}(x_o, \dot{x}_o) &\equiv -\gamma \frac{\partial \dot{Q}}{\partial S_{\mathrm{damp}}} = -\gamma (H(x_o, \dot{x}_o) - H_o)\dot{x}_o \\ &= -\frac{\gamma}{2}(m_o \dot{x}_o^2 + c_o x_o^2 - 2H_o)\dot{x}_o \end{aligned} \tag{4}$$

with the gain parameter $\gamma > 0$ can be used to achieve asymptotically stable periodic motions of the unforced system. Due to the special form of differential equation (2) these periodic motions are harmonic.

In case of the harmonically forced system, the state-dependent force

$$S(x_o, \dot{x}_o) = -\frac{\gamma}{2}(m_o \dot{x}_o^2 + \nu m_o x_o^2 - 2H_o)\dot{x}_o - c_o x_o \tag{5}$$

with the additional, yet unknown factor $\nu > 0$, can lead to an asymptotically stable harmonic response, if the parameters of the oscillator ν, H_o, c_o are adapted to the excitation, see [6] for details. The equation of motion of the excited system according to Fig. 1 then becomes

$$m_o \ddot{x}_o + \frac{\gamma}{2}(m_o \dot{x}_o^2 + \nu m_o x_o^2 - 2H_o)\dot{x}_o + c_o x_o = m_o \underbrace{\Omega^2 \hat{x}_1 \cos(\Omega t + \alpha_1)}_{-\ddot{x}_1}. \tag{6}$$

It describes a driven Rayleigh–van der Pol oscillator with in general periodic but non-harmonic response.

In [6] it was shown that the parameters of the oscillator ν, H_o, c_o can be adapted to the base excitation $x_1(t)$ with arbitrary phase angle α_1 in such a manner that the 1/1-synchronization has the exact steady state response $x_o(t) = \hat{x}_o \cos \Omega t$, where the amplitude $\hat{x}_o$ is a free parameter. Introducing the desired harmonic response $x_o(t) = \hat{x}_o \cos \Omega t$ into (6) and balancing of the coefficients of the harmonics yields

$$0 = a_1 \sin \Omega t + b_1 \cos \Omega t + a_3 \sin 3\Omega t + b_3 \cos 3\Omega t \tag{7}$$

with the coefficients of the harmonics

$$\left.\begin{aligned} a_1 &= -\frac{3\,\Omega^3\,\gamma\, m_o\, \hat{x}_o^3}{8} + m_o \Omega^2 \hat{x}_1 \sin \alpha_1 \\ &\quad + \frac{\Omega\left(8\,H_o\,\gamma\,\hat{x}_o - \gamma\, m_o\, \nu\, \hat{x}_o^3\right)}{8}, \\ b_1 &= c_o\,\hat{x}_o - m_o\,\Omega^2\,\hat{x}_o - m_o \Omega^2 \hat{x}_1 \cos \alpha_1, \\ a_3 &= -\frac{\Omega\,\gamma\, m_o\, \hat{x}_o^3\left(\nu - \Omega^2\right)}{8}, \\ b_3 &= 0. \end{aligned}\right\} \tag{8}$$

If the coefficients (8) vanish, the system has a harmonic response under 1/1-synchronization. This is fulfilled for the parameters

$$\nu = \Omega^2, \tag{9}$$

$$c_o = m_o\,\Omega^2\,\frac{\hat{x}_o + \hat{x}_1 \cos \alpha_1}{\hat{x}_o}, \tag{10}$$

and

$$H_o = m_o\,\frac{\gamma\,\Omega^2\,\hat{x}_o^3 - 2\Omega \hat{x}_1 \sin \alpha_1}{2\gamma\,\hat{x}_o} \tag{11}$$

Introducing the solutions for the parameters (9) to (11) into (6) yields the differential equation of the RvdP oscillator with harmonic response under 1/1-synchronization,

$$\begin{aligned} m_o \ddot{x}_o + \frac{\gamma}{2}\Big(m_o \dot{x}_o^2 + \Omega^2 m_o x_o^2 - \overbrace{m_o\,\frac{\gamma\,\Omega^2\,\hat{x}_o^3 - 2\,\Omega \hat{x}_1 \sin \alpha_1}{\gamma\,\hat{x}_o}}^{2H_o}\Big)\dot{x}_o \\ + \underbrace{m_o\,\Omega^2\,\frac{\hat{x}_o + \hat{x}_1 \cos \alpha_1}{\hat{x}_o}}_{c_o} x_o \\ = m_o\,\Omega^2 \hat{x}_1 \cos(\Omega t + \alpha_1). \end{aligned} \tag{12}$$

Since system (12) shows 1/1-synchronization for arbitrary values of $\hat{x}_o, \hat{x}_1, \Omega$ and α_1, the solutions for the parameters (9) to (11) are called the *existence conditions* for 1/1-synchronization.

To study the stability of 1/1-synchronization, the nonlinear equation of the oscillator (12) is harmonically linearized into the form

$$m_o \ddot{x}_o + d_o \dot{x}_o + c_o x_o = m_o \Omega^2 \hat{x}_1 \cos(\Omega t + \alpha_1) \tag{13}$$