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Three-dimensional vibration of cantilevered fluid-conveying micropipes—Types of periodic motions and small-scale effect

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A B S T R A C T

A new theoretical model is developed for the three-dimensional (3D) nonlinear vibration analysis of fluid-conveying cantilevered micropipes. Particular attention is given on the derivation and analysis of the reduced equations, and the small-scale effect on the periodic motions. Based on the modified couple stress theory (MCST), the governing equations are derived by using Hamilton's principle. The material length scale parameter and large-deflection-induced geometric nonlinearities given by the Lagrangian strain tensor are incorporated into the governing equations. Utilizing the center manifold theory, normal form method and O(2) symmetry, the original governing equations can be rigorously reduced to a two-degree-of-freedom (2DOF) dynamical system. Then two possible types of periodic motions, i.e. planar periodic and spatial periodic motions, together with their stabilities are investigated by means of averaging methods and numerical simulations. Results show that the larger the dimensionless material length scale parameter is, the wider the region of mass ratio for stable planar periodic motion is. Particularly, the presence of small length scale parameter makes micropipes be more likely to oscillate in a plane. It is also shown that for mass ratio corresponding to the hysteresis of the curves of critical flow velocity versus mass ratio, the stabilities for bifurcating periodic motions at lower, moderate and higher critical flow velocities may be different.

1. Introduction

Cantilevered pipes conveying fluid have been studied extensively [1–9]. The literature on the nonlinear dynamics of cantilevered pipes was mainly concerned with two-dimensional (2D) models. However, the literature concerning the three-dimensional (3D) oscillations of fluid-conveying cantilevered pipes is relatively limited. The earliest work contributed to the 3D models of fluid-conveying cantilevered pipes is due to Lundgren et al. [10], who derived a 3D version of nonlinear governing equations by using the force balance method. By means of the center manifold and normal form techniques, Bajaj et al. [11] found that cantilevered pipes conveying fluid can develop either 2D or 3D limit cycle motion after losing its original stability through a supercritical Hopf bifurcation, showing that the type of oscillations depends on the mass ratio parameter β [defined later, in Eq. (30)]. Indeed, in the past several years, a few papers have dealt with the 3D motions of cantilevered pipes conveying fluid. Using the modified Hamilton principle developed by Benjamin [8], Wadham-Gagnon et al. [12] derived a set of 3D nonlinear equations for a cantilevered pipe conveying fluid in the presence of an additional mass or spring attached to it. Based on this model, the 3D motion of a cantilevered pipe conveying fluid with an end-mass [13], with an added spring [14], or with both an end-mass and an added spring [15] have been studied. If an additional mass is attached to the end of the pipe, the resulting dynamics becomes much richer than that of pipes without any external attachments. It was found that for very large end-mass, a large number of Galerkin's truncated modes are required to obtain convergent results [16]. The results reported in [14] showed that a cantilevered pipe with an external spring along its length would exhibit 2D or 3D periodic, quasiperiodic and chaotic oscillations beyond the onset of flutter. Compared to the previous study by Païdoussis et al. [14], a more complete, accurate and interesting work was done by Ghayesh et al. [17], who investigated the role of spring configuration and its location along the pipe length. Chang et al. [18] extended Wadham-Gagnon et al.'s equations [12] by introducing a base excitation, and applied them to investigate the

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possibility of controlling the pipe's 3D motion and/or limiting it to a 2D motion in a pre-defined direction, by changing the frequencies and amplitudes of base excitation.

Due to recent technological developments in micro-engineering, the characteristic size of pipes becomes smaller and smaller. Miniaturized beams/pipes have become one of the core components of micro-electronic–mechanical-systems (MEMS) [19–21] and magneto-electro-elastic-systems (MEES) [22]. In 2010, the dynamics of microscale pipes containing internal fluid flow have been studied by Rinaldi et al. [23] in the context of the classical continuum mechanics theory, where the inside diameter of the circular micropipe ranges from 1 to 100 μm. Recently, size-dependent behaviors of microscale structures have been observed experimentally {see, e.g., Fleck et al. [24], Lam et al. [25], McFarland and Colton [26]}. In many cases, therefore, we cannot directly extend the analysis of macroscale structures to that of microscale structures. For that reason, several non-classical elasticity theories, such as the modified strain gradient theory (MSGT) and modified couple stress theory (MCST) [27], have been introduced to study the behavior of microscale structures by incorporating size dependence. For bending and torsion behaviors of microscale structures, as discussed by Xu et al. [28], the MCST is more adequate for describing the size-dependent effect.

For fluid-conveying micropipes with both ends supported, a theoretical model was developed by Wang [29] for the linear vibration analysis, in which the Euler–Bernoulli beam assumption and the MCST were employed. In another work by Xia and Wang [30], the size-dependent vibration of micropipes was analyzed using Timoshenko beam models. Yang et al. [31] investigated the microfluid-induced nonlinear free vibration of micropipes with both ends immovable by using the MCST. The geometric nonlinearity arising from the mid-plane stretching was taken into account and the static post-buckling problem was also discussed. Mashroutech et al. [32] utilized the same nonlinear equation of motion and revisited the nonlinear frequencies based on a three-mode approximation of Galerkin's approach and the variational iteration method. Tang et al. [33] have developed a nonlinear theoretical model for size-dependent 3D vibration analysis of curved micropipes conveying fluid with clamped–clamped ends based on the MCST. The Lagrangian nonlinear axial strain was adopted to obtain the static deformation induced by the internal fluid flow. Wang et al. [34] investigated the dynamics of microscale pipes conveying fluid with consideration of size effects of both micro-flow and micro-structure, for either straight or curved pipes with cross-section of internal fluid devised as circular, elliptic or rectangular shapes. In the work by Farokhi et al. [35], molecular dynamics simulations were performed for the analysis of carbon nanotube-based resonators. The validity of the classical continuum mechanics theory and the developed size-dependent continuum model at the nanoscale was checked.

Perhaps the first study of the dynamics of cantilevered micropipes conveying fluid is contributed by Hosseini and Bahaadini [36], who derived the linear equation of motion based on the MSGT and then performed an analysis of eigenvalues with a parametric study to examine the effect of length scale parameter. In another paper of Bahaadini and Hosseini [37], the effect of fluid slip condition on the free vibration and flutter instability of viscoelastic cantilevered carbon nanotubes (CNTs) conveying fluid were investigated. The material property of the CNT was simulated by the Kelvin–Voigt viscoelastic constitutive relation. The equations derived by Hosseini and Bahaadini are linear. Hu et al. [38] developed a nonlinear 2D model for cantilevered micropipes conveying fluid and explore the possible size-dependent nonlinear responses based on the MCST. To the author's knowledge, however, the literature on nonlinear modeling and nonlinear equations of motions for cantilevered micropipes with consideration of small length scale effect is very limited and hence the 3D nonlinear dynamics of this system have not been reported. This motivates the current work.

The objective of this study is to develop a microstructure-dependent 3D nonlinear model and apply it to investigate the 2D and 3D periodic motions of fluid-conveying cantilevered micropipes. Attention is focused on the effect of small length scale on the two types of periodic motions. The paper is organized as follows. In Section 2, based on the geometrical analysis of 3D motions of the cantilevered micropipe conveying fluid and the MCST, the 3D version of governing equations are derived, in which the material length scale parameter is incorporated. Utilizing the center manifold theory, normal form method and O(2) symmetry, the original governing equations are rigorously reduced to a two-degree-of-freedom (2DOF) vibration system in Section 3. Section 4 deals with two possible types of periodic motions and their stabilities by means of the averaging methods and numerical simulations. Some conclusions are drawn out in Section 5.

2. Derivation of the equations of motions

The system under consideration consists of a uniform micropipe of length L with circular cross-section, external cross-sectional area A_p, mass m per unit length, mass ρ per unit volume, conveying incompressible fluid of mass M per unit length, flowing axially with velocity V not varying with time; see Fig. 1. Here we introduce both Lagrangian coordinate system (X, Y, Z) and Eulerian coordinate system (x, y, z) in the same way as that proposed by Wadham-Gagnon et al. [12].

As we know, the 3D version of nonlinear equations of motion for a macro cantilevered pipe conveying fluid has been derived by Wadham-Gagnon et al. [12], who defined the strain energy by an expression related to the curvature of the pipe. For micropipes, in the presence of size-dependent behavior, the expression of strain energy for macropipes cannot be directly applied. According to the modified couple stress formulation [27], the displacements $u_1(X,Y,Z,t), u_2(X,Y,Z,t)$ and $u_3(X,Y,Z,t)$ of any material point of the pipe at moment t in the x, y and z directions, respectively, are required to derive the formula of strain energy. It should be mentioned that X, Y and Z are the Lagrangian coordinates introduced to label particles of the pipe at the original equilibrium state, and they are related to the Eulerian coordinates x, y and z as

$$u_1(X,Y,Z,t) = x - X, u_2(X,Y,Z,t) = y - Y, u_3(X,Y,Z,t) = z - Z. \quad (1)$$

According to the MCST, the strain energy U in a deformed isotropic linear elastic material occupying region Ω can be written as [27]

$$U = \frac{1}{2}\int_{\Omega}(\boldsymbol{\sigma} : \boldsymbol{\varepsilon} + \mathbf{m} : \boldsymbol{\chi})\mathrm{d}v. \quad (2)$$

Unless otherwise specified, we denote $\mathrm{d}v = \mathrm{d}X\mathrm{d}Y\mathrm{d}Z$. In Eq. (2), the stress tensor $\boldsymbol{\sigma}$, the strain tensor $\boldsymbol{\varepsilon}$, the deviatoric part of the couple stress tensor $\mathbf{m}$, and the symmetric curvature tensor $\boldsymbol{\chi}$, are given by

$$\boldsymbol{\sigma} = \lambda \mathrm{tr}(\boldsymbol{\varepsilon})\boldsymbol{\delta} + 2G\boldsymbol{\varepsilon} \quad (3)$$

$$\boldsymbol{\varepsilon} = (1/2)[\nabla\mathbf{u} + (\nabla\mathbf{u})^{\mathrm{T}}] + (1/2)\nabla\mathbf{u}\cdot(\nabla\mathbf{u})^{\mathrm{T}} \quad (4)$$

$$\mathbf{m} = 2l^2 G\boldsymbol{\chi} \quad (5)$$

$$\boldsymbol{\chi} = (1/2)[\nabla\boldsymbol{\theta} + (\nabla\boldsymbol{\theta})^{\mathrm{T}}] \quad (6)$$

respectively. In Eqs. (3)–(6), λ and G are the Lamé's constants, $\boldsymbol{\delta}$ is the Kronecker's tensor, Eq. (4) is the Lagrangian strain tensor representing the large-deflection-induced nonlinearities, ∇ is the Lagrangian gradient operator. l is a material length scale parameter [27]. Generally, different materials have different values of l, and $l = 0$ is for macropipes [29]. In Eq. (4), $\mathbf{u}$ is the displacement vector with components $u_1(X,Y,Z,t), u_2(X,Y,Z,t)$, and $u_3(X,Y,Z,t)$. In Eq. (6), $\boldsymbol{\theta}$ is the rotation vector and is given by

$$\boldsymbol{\theta} = (1/2)\mathrm{curl}(\mathbf{u}). \quad (7)$$

In the following, a curvilinear coordinate s, along the length of the deformed pipe is introduced. In fact, s is equal to X [12]. According to the Euler–Bernoulli beam assumption, a circular cross-section lying in the $\mathbf{j}'\mathbf{k}'$ plane [see Fig. 2(d)] is considered as a rigid one, i.e., there is no deformation during oscillations, which means that the displacement

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