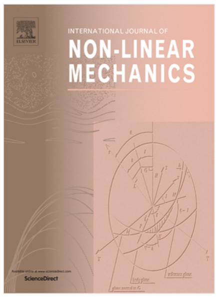
Accepted Manuscript



A non-ellipticity result, or the impossible taming of the logarithmic strain measure

Robert J. Martin, Ionel-Dumitrel Ghiba, Patrizio Neff

PII: S0020-7462(17)30829-6
DOI: https://doi.org/10.1016/j.ijnonlinmec.2018.02.011
Reference: NLM 2981

To appear in: *International Journal of Non-Linear Mechanics*

Received date : 13 December 2017
Revised date : 16 February 2018
Accepted date : 21 February 2018

A non-ellipticity result, or the impossible taming of the logarithmic strain measure

Robert J. Martin[1] and Ionel-Dumitrel Ghiba[2] and Patrizio Neff[3]

February 15, 2018

Abstract

Constitutive laws in terms of the logarithmic strain tensor $\log U$, i.e. the principal matrix logarithm of the stretch tensor $U = \sqrt{F^TF}$ corresponding to the deformation gradient F, have been a subject of interest in nonlinear elasticity theory for a long time. In particular, there have been multiple attempts to derive a viable constitutive law of nonlinear elasticity from an elastic energy potential which depends solely on the logarithmic strain measure $\|\log U\|^2$, i.e. an energy function $W\colon \mathrm{GL}^+(n) \to \mathbb{R}$ of the form

$$W(F) = \Psi(\|\log U\|^2) \tag{1}$$

with a suitable function $\Psi\colon [0,\infty) \to \mathbb{R}$, where $\|\,.\,\|$ denotes the Frobenius matrix norm and $\mathrm{GL}^+(n)$ is the group of invertible matrices with positive determinant.

However, while such energy functions enjoy a number of favorable properties, we show that it is not possible to find a strictly monotone function Ψ such that W of the form (1) is Legendre-Hadamard elliptic.

Similarly, we consider the related isochoric strain measure $\|\mathrm{dev}_n \log U\|^2$, where $\mathrm{dev}_n \log U$ is the deviatoric part of $\log U$. Although a polyconvex energy function in terms of this strain measure has recently been constructed in the planar case $n = 2$, we show that for $n \geq 3$, no strictly monotone function $\Psi\colon [0,\infty) \to \mathbb{R}$ exists such that $F \mapsto \Psi(\|\mathrm{dev}_n \log U\|^2)$ is polyconvex or even rank-one convex. Moreover, a volumetric-isochorically decoupled energy of the form $F \mapsto \Psi(\|\mathrm{dev}_n \log U\|^2) + W_{\mathrm{vol}}(\det F)$ cannot be rank-one convex for any function $W_{\mathrm{vol}}\colon (0,\infty) \to \mathbb{R}$ if Ψ is strictly monotone.

Contents

[1]Corresponding author: Robert J. Martin, Lehrstuhl für Nichtlineare Analysis und Modellierung, Fakultät für Mathematik, Universität Duisburg-Essen, Thea-Leymann Str. 9, 45127 Essen, Germany; email: robert.martin@uni-due.de

[2]Ionel-Dumitrel Ghiba, Alexandru Ioan Cuza University of Iaşi, Department of Mathematics, Blvd. Carol I, no. 11, 700506 Iaşi, Romania; Octav Mayer Institute of Mathematics of the Romanian Academy, Iaşi Branch, 700505 Iaşi and Lehrstuhl für Nichtlineare Analysis und Modellierung, Fakultät für Mathematik, Universität Duisburg-Essen, Thea-Leymann Str. 9, 45127 Essen, Germany; email: dumitrel.ghiba@uni-due.de, dumitrel.ghiba@uaic.ro

[3]Patrizio Neff, Head of Lehrstuhl für Nichtlineare Analysis und Modellierung, Fakultät für Mathematik, Universität Duisburg-Essen, Thea-Leymann Str. 9, 45127 Essen, Germany; email: patrizio.neff@uni-due.de