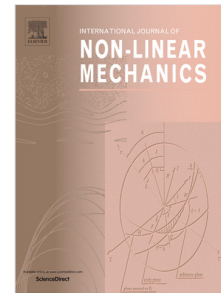


## Accepted Manuscript

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PII: S0020-7462(17)30829-6  
DOI: <https://doi.org/10.1016/j.ijnonlinmec.2018.02.011>  
Reference: NLM 2981

To appear in: *International Journal of Non-Linear Mechanics*

Received date: 13 December 2017  
Revised date: 16 February 2018  
Accepted date: 21 February 2018

Please cite this article as: R.J. Martin, I.-D. Ghiba, P. Neff, A non-ellipticity result, or the impossible taming of the logarithmic strain measure, *International Journal of Non-Linear Mechanics* (2018), <https://doi.org/10.1016/j.ijnonlinmec.2018.02.011>

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# A non-ellipticity result, or the impossible taming of the logarithmic strain measure

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February 15, 2018

## Abstract

Constitutive laws in terms of the logarithmic strain tensor  $\log U$ , i.e. the principal matrix logarithm of the stretch tensor  $U = \sqrt{F^T F}$  corresponding to the deformation gradient  $F$ , have been a subject of interest in nonlinear elasticity theory for a long time. In particular, there have been multiple attempts to derive a viable constitutive law of nonlinear elasticity from an elastic energy potential which depends solely on the logarithmic strain measure  $\|\log U\|^2$ , i.e. an energy function  $W: \text{GL}^+(n) \rightarrow \mathbb{R}$  of the form

$$W(F) = \Psi(\|\log U\|^2) \quad (1)$$

with a suitable function  $\Psi: [0, \infty) \rightarrow \mathbb{R}$ , where  $\|\cdot\|$  denotes the Frobenius matrix norm and  $\text{GL}^+(n)$  is the group of invertible matrices with positive determinant.

However, while such energy functions enjoy a number of favorable properties, we show that it is not possible to find a strictly monotone function  $\Psi$  such that  $W$  of the form (1) is Legendre-Hadamard elliptic.

Similarly, we consider the related isochoric strain measure  $\|\text{dev}_n \log U\|^2$ , where  $\text{dev}_n \log U$  is the deviatoric part of  $\log U$ . Although a polyconvex energy function in terms of this strain measure has recently been constructed in the planar case  $n = 2$ , we show that for  $n \geq 3$ , no strictly monotone function  $\Psi: [0, \infty) \rightarrow \mathbb{R}$  exists such that  $F \mapsto \Psi(\|\text{dev}_n \log U\|^2)$  is polyconvex or even rank-one convex. Moreover, a volumetric-isochorically decoupled energy of the form  $F \mapsto \Psi(\|\text{dev}_n \log U\|^2) + W_{\text{vol}}(\det F)$  cannot be rank-one convex for any function  $W_{\text{vol}}: (0, \infty) \rightarrow \mathbb{R}$  if  $\Psi$  is strictly monotone.

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