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Bifurcation analysis of an accelerating disc immersed in a bounded compressible medium near principal parametric resonance

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ABSTRACT

This paper investigates the effect of low frequency parametric excitations on the coupled dynamics of a spinning disc bounded in a compressible fluid filled enclosure. Specifically, the effect of principal parametric resonance due to disc accelerations is studied. The angular acceleration of the disc results in the governing equations being represented as nonlinearly coupled non-autonomous gyroscopic system. Investigations into the bifurcation behaviour is first studied using Floquet theory on the linearised equations; the stable and unstable regions of the trivial solution are obtained in the amplitude–frequency parameter space corresponding to the acceleration, for various damping configurations. A codimension-two bifurcation is observed when two control parameters are considered in the bifurcation analysis. The bifurcation behaviour subsequently investigated through direct numerical integration of the governing equations, reveals that the coupled system exhibits a two-period quasi periodic oscillation in the post bifurcation regime.

1. Introduction

Study of the aeroelastic instabilities of a spinning disc immersed in a compressible fluid medium is very important from the design point of view for many engineering systems, such as saw blades, solar sails, computer hard discs and turbo-machinery components. These instabilities are due to the dynamic interaction between a flexural wave of the disc and an acoustic wave of the surrounding compressible fluid. Aeroelastic instabilities induce large amplitude oscillations in both the structure and the surrounding acoustic fluid and can cause deviation in the performance of the system, reduce the overall efficiency, and in extreme cases may lead to system failure. Angelo et al. [1] were one of the first to study the aeroelastic instability mechanisms of a rotating disc housed in bounded fluid environment experimentally. By means of the Moiré photograph, they showed that during flutter instability, single mode travelling wave [2] dominates the overall response. They also measured the flutter speed and showed the importance of fluid density. Renshaw et al. [3] studied the aeroelastic instability mechanisms of a rotating disc coupled to the surrounding bounded acoustic fluid, both analytically and experimentally. Through their experiments, the significance of the fluid density on the flutter speed was demonstrated. In their analytical model, they used a compressible potential flow theory with the solution of the wave equation using Hankel transforms for the aerodynamic load estimation on the disc. These studies [1,3] did not provide any clear insights about the disc–fluid coupling or the instability mechanisms. In order to throw some light on the physics of the coupled problem, Kang and Raman [4,5] developed an analytical framework based on the theoretical formulation of Dowell et al. [6]. By solving the coupled eigenvalue problem, they showed that there exists three instability mechanisms with respect to the system damping. Their model can be used to predict the flutter onset speeds, the fluttering mode and the instability regions. However, as their model is based on linear elastic theory, the predictions for the post bifurcation dynamics are unrealistic. Recently, Dheelibun et al. [7] extended this study by including the geometric nonlinearities that arise due to large oscillations to study the post flutter dynamics as well. Bifurcation analysis of the resulting nonlinear gyroscopically coupled autonomous system revealed that depending upon the system damping, the route to instability could be either through Hamiltonian Hopf, degenerate Hopf or supercritical Hopf bifurcation.

It is important to note that this analysis [7] considered the disc to be rotating with a constant spin rate. In fact, all the studies discussed so far investigated the aeroelastic instabilities of a disc spinning at a constant rate in the surrounding fluid. In reality, fluctuations in the spin rate of the disc can be triggered by various factors such as power fluctuations, wear and tear in the bearings of the shaft attached to

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the disc, during start-up or shut down, as well as, due to other forms of imperfections—manufactured or otherwise. This non-constant spin rate enters into the governing equations as parametric excitation [8,9]. This study investigates the effect of this non-constant spin rate on the aeroelastic instability mechanisms.

Studies on the transverse dynamics of a disc spinning at non-constant spin rate appear to have not received much attention in the literature. Malhotra and Namachchivaya [10] performed a local bifurcation analysis of a spinning disc *in vacuo* subjected to non-constant spin rate. Here, by assuming a harmonic variation for the spin rate, the authors investigated the sub-harmonic response of the disc system. The resulting non-autonomous equations of motion were transformed into the autonomous form using the method of time averaging [11]. Subsequently, the authors analysed the local stability of the resulting equilibrium solutions using conventional linear stability theory with the detuning parameter as a bifurcation parameter. The detuning parameter expresses the proximity of the excitation frequency to the system natural frequency. The study revealed that the equilibrium solution can manifest a pitchfork bifurcation. The effect of imperfections on the local dynamics were investigated as well. Later, Malhotra et al. [12] extended this study to analyse the global dynamics of the same spinning disc system in the *in vacuo* condition. By employing the energy-phase method [13], the authors proved the existence of single-pulse and multi-pulse orbits to equilibrium solutions. The authors concluded that the existence of such orbits implies the existence of chaos in the system. Hansen [9] had also reported the linear transverse dynamics of a spinning disc in the absence of fluid, subjected to non-constant spin rate. In this case also, a harmonic variation for the spin rate was assumed and the frequency of the spin rate was considered to be significantly higher compared to the lower mode natural frequencies of the system. These parametric excitations are therefore referred to as fast excitations [14]. Using the Modified Method of Multiple Scales (MMMS) [14], the effects of the fast excitation on the linear dynamics were studied. It was shown that the effect of fast parametric excitation results in positive definite vibrational force, and as a result, the natural frequencies of the lower asymmetrical modes were seen to increase with the forcing amplitude. This work also revealed the stiffening effect of the fast parametric excitation. It must be emphasised here once again that all these studies were undertaken for disc systems only, without considering the effect of any fluid environment.

Investigations on the effect of the higher order (fast) parametric excitations for a disc–fluid system on the aeroelastic instability mechanisms has been carried out in an earlier study by Dheelibun et al. [15]. In [15], the disc is considered to be immersed in a bounded fluid medium and the resultant governing equations consider the strong acousto-coupling effects. It was observed that inclusion of the fast excitation leads to natural separation of the system motion into fast and slow components. The resulting non-autonomous equations of the system motion were transformed into the autonomous equation governing the slow components using MMMS and the time averaging techniques. Bifurcation analysis had been performed on the autonomous equations for various dissipation configurations. From this study, the authors concluded that the stiffening and gyroscopic effects of the fast parametric excitations can postpone or suppress the aeroelastic flutter instability of the coupled disc–fluid system.

Unlike [15] where the parametric excitation frequencies were assumed to be significantly higher than the frequencies of the natural modes of the system, the present study considers the case when the frequency of the temporal variations are of the same order as the lower mode natural frequencies of the system. In keeping with the terminology of "fast excitation" for the earlier case [15], these excitations can be referred to as lower order excitations. Specifically, this study investigates the coupled dynamics by choosing the frequency of the time-varying part in such a way that it is nearly twice the natural frequency of the disc *in vacuo* modes. This condition is reported in the literature as principal parametric resonance [11] and the corresponding response is known as sub-harmonic response [10]. As mentioned earlier, [10,12] studied the sub-harmonic resonance of a spinning disc *in vacuo* for both perfect and imperfect configurations. To the best of the authors' knowledge, there appears to be no studies available on the bifurcation dynamics of the disc–fluid system with non-constant spin rate, subjected to principal parametric resonance.

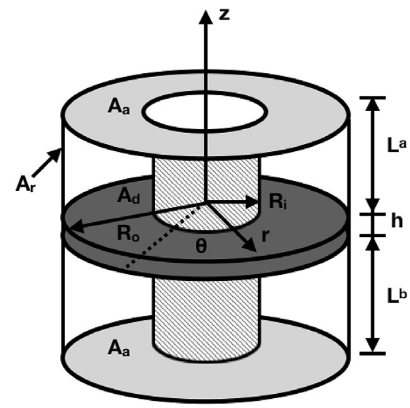

Fig. 1. Schematic of a disc spinning at a non-constant speed in compressible fluid filled enclosure. Here, A_a, A_r and A_d respectively denote the absorbent wall, rigid wall and the disc surface areas.

Since the dimension (with respect to the number of modes considered for computation) of the coupled system is significantly higher as compared to the disc system considered in [10], the use of approximate techniques such as method of averaging, is cumbersome and infeasible. Hence, Floquet theory [16,17] and direct numerical integration (DNI) have been used for the bifurcation analysis in the present study. The Floquet theory is used to predict the stability of the equilibrium and/or periodic solutions as well as to predict the onset of instability. The DNI provides the details of the post bifurcation dynamics of the coupled system.

The paper is organised as follows: Section 2 details the system being studied and presents the corresponding mathematical formulation. Section 3 focusses on the bifurcation analysis of the disc–fluid system using Floquet theory as well as DNI. The paper concludes in Section 4 with a discussion on the primary findings from this study. An Appendix is provided at the end which gives details of some of the mathematical equations used in the formulation presented in Section 2.

2. Formulation

Consider an annular disc of thickness h, fixed at inner radius R_i and free at outer radius R_o rotating about its axis of symmetry in a compressible fluid filled enclosure; see Fig. 1 for a schematic diagram of the system. The disc is assumed to be rotating with a non-constant spin rate, $\Omega_d = \Omega_m(1+\Omega_n(t))$, where, Ω_m is the mean angular velocity, $\Omega_n(t) = A\sin\Omega_v t$ is the time varying part of the rotation rate with $\Omega_m A$ being the amplitude of the temporal fluctuations, t is the time and Ω_v is the excitation frequency.

In order to obtain the field equations in the non-dimensional form, the field variables of the acceleration parameters are non-dimensionalised as $\Omega_0 = \Omega_m t_0$, $\tau = t/t_0$, $a = \Omega_m A t_0$ and $\lambda = \Omega_v t_0$, where, Ω_0 is the non-dimensional mean speed, τ is the non-dimensional time, a is the non-dimensional amplitude and λ is the non-dimensional frequency. The other field variables are non-dimensionalised as in [7]. Here, t_0 is a characteristic time defined as the time taken by a flexural wave of a stationary disc to travel the wavelength of $2\pi R_o$. Thus, the rotational speed Ω_d is written in the non-dimensional form as $\Omega = \Omega_0 + \Omega_1(\tau) = \Omega_0 + a\sin\lambda\tau$.

The field equations governing the interaction of the disc–fluid system subjected to non-constant spin rate Ω, with respect to the ground fixed