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Bifurcations of periodic motion of a horizontally supported nonlinear Jeffcott rotor system having transversely cracked shaft

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ABSTRACT

This article investigates the periodic motion bifurcations of a horizontally supported nonlinear Jeffcott rotor system having transversely cracked shaft. The nonlinear spring characteristics due to Hertz contact force and bearing clearance, disc weight, disc eccentricity, breathing of the shaft crack, and angle between the crack and imbalance directions are included in the system model. A mathematical model governing the cracked system lateral vibrations is derived and then analyzed utilizing asymptotic analysis in the primary resonance case. Effects of disc eccentricity, creak depth, and angle between the crack and imbalance directions on the system response curves are studied. The analysis revealed that at a small crack depth, the system executes both forward and backward whirling motions at a specific range of the disc spinning speed, while the backward whirling orbits disappear as the crack depth increases. In addition, at zero disc eccentricity, the cracked system does not oscillate unless the system linear stiffness coefficient is reduced by about 11% as a result of shaft crack. Moreover, there is a spinning speed range of the rotating shaft at which two stable periodic solution attractors appear beside the trivial solution one when the linear stiffness coefficient of the system is reduced to 20% or more. The obtained analytical results are confirmed numerically that showed a very good agreement with the numerical ones. Finally, the acquired results are compared with the work published in the literature.

1. Introduction

Rotating machinery has an important role in modern industries because of their massive applications such as generators, aerospace, steam and gas turbines, automobile engines, turbomachinery, pumps, high-speed compressors, in addition to their domestic applications. The rotating machinery vibration is an inherent phenomenon, which arises due to the mass imbalance, the dynamic interaction between the stator and rotating parts, and the shaft cracks. The existence of cracks in the rotating machinery shaft may be one of the most serious reasons for its damage, catastrophic failures and dangerous accidents in aircraft engines and other rotating machinery. Accordingly, studying the dynamical behaviors of the cracked rotors has received considerable attention of many researchers in the last four decades. Wauer [1] and Dimarogonas [2] introduced comprehensive reviews on the dynamical behaviors of the cracked Jeffcott rotor system. They reported many dynamical phenomena that could be used as diagnostic tools for the presence of cracks on the rotating machinery. Gasch [3] investigated the oscillatory behaviors of cracked Laval-rotor model, where the simple hinge model is employed to represent the cyclic stiffness variables of a cracked solid shaft. The author reported that 1:3 and 1:2 superharmonic resonances are excited in addition to the primary resonance case. Jun et al. [4] studied the nonlinear vibrations of a simple Jeffcott rotor system having transversely cracked shaft. They employed the switching crack model to represent the cyclic stiffness variables of the cracked shaft. They showed that the second horizontal harmonic component measured near the second harmonic resonant speed is a clear indication for the existence of the shaft crack. In Ref. [5] Sinou et al. utilized the alternate frequency/time domain approach to explore the cracked rotor system dynamics. They found that the change of dynamical characteristics of the rotor system near half of resonant speed is a positive indication of the presence of shaft crack. Sinou et al. [6,7] presented a nonlinear study to the rotor system having a transverse cracked shaft. He demonstrated that the cracked system performs a complicated response at 1:2 and 1:3 superharmonic resonance and the shaft executes two and three whirling loops per shaft revolution. Moreover, the cracked shaft may lose its stability at a specific crack depth. Sinuous [8] studied the cracked rotor system with crack breathing model numerically. He illustrated that the evolution of the superharmonic resonance component

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Nomenclature

$y_1, \dot{y}_1, \ddot{y}_1$	Dimensionless displacement, velocity, and acceleration of the disc geometric center in Y_1-direction.
$y_2, \dot{y}_2, \ddot{y}_2$	Dimensionless displacement, velocity, and acceleration of the disc geometric center in Y_2-direction.
a_1, a_2	Dimensionless whirling amplitudes of the disc geometric center in Y_1 and Y_2-directions, respectively.
μ_1, μ_2	Dimensionless linear damping coefficients in Y_1 and Y_2-directions, respectively.
ω_1, ω_2	Dimensionless linear natural frequency of the cracked system in Y_1 and Y_2-directions, respectively.
δ	Dimensionless cubic and quadratic nonlinearities coefficient.
Ω	Dimensionless disc spinning speed.
E	Dimensionless disc-eccentricity magnitude.
λ	Dimensionless parameter representing the relative reduction of the shaft linear stiffness coefficient due to the crack.
γ	Denotes the angle between the crack and imbalance directions

that was excited at 1:2 and 1:3 resonance cases could be used to detect the cracks in the rotating machinery. Chen et al. [9] established a new method to analyze the dynamical behaviors of a nonlinear cracked rotor system. They illustrated that their proposed approach can investigate efficiently the cracked rotor system with considerations of the disc spinning speed, disc mass, and shaft geometric nonlinearity. Then, the authors approved that the crack depth has a negligible effect on the primary response as long as the crack depth is lower than half the shaft cross-sectional area, while small crack depths have significant effects at superharmonic resonance cases. Han et al. [10] discussed the influence of double cracks on the dynamical behavior of the Jeffcott rotor system. The authors concluded that the existence of two cracks on the rotor shaft has a significant effect on the system response that differ completely that of one crack system. The nonlinear dynamics of a cracked rotor system having asymmetrical viscoelastic supports are introduced in Ref. [11]. The authors explored the influences of the different system parameters such as the crack-depth, crack position, disc position and disc thickness using the harmonic balance method.

In Ref. [12] Lin et al. discussed the torsional excitation on the cracked rotor system. AL-Shudeifat et al. [13] utilized harmonic balance method and breathing crack mechanism to explore the dynamic characteristics of a cracked rotor system. They found that the obtained theoretical results are in excellent agreements with the practical results when the length and diameter of the shaft are too large and the crack thickness is narrow such that the crack breathing becomes more similar to the theoretical model. Jun et al. [14,15] analyzed the dynamic behaviors of the cracked rotor system using additional slope and bending moment at the crack position. Ishida [16] investigated different resonant cases of the cracked Jeffcott rotor system. Cheng et al. [17] discussed the influences of the angle between the crack and imbalance directions on the vibration level of the cracked Jeffcott rotor system at synchronous whirling. They concluded that the maximum vibration peak occurs when the angle is zero, while the minimum vibration peak happens at an angle equals to π. Hou et al. [18] studied a cracked Jeffcott rotor system having Duffing type nonlinearity and simulating aircraft-rotor subjected to maneuver loads. The authors applied the multiple scales perturbation method to study the local bifurcations of the system at 1:2 and 1:3 superharmonic resonance cases. Hou et al. [19] presented a numerical study for the dynamical behaviors of the cracked rotor-ball bearing system caused by aircraft flight maneuvers. The influences of maneuver load, crack stiffness, bearing clearance, and rotor eccentricity on the system responses are investigated. They reported that quasi-periodic motions have been noticed, and a variety of complex nonlinear behaviors including bifurcations and jumping phenomenon have been observed near 1/4, 1/3, 2/5 and 1/2 critical speed when the maneuver load increases. Hou et al. [20] investigated the nonlinear vibration of the cracked rotor system at superharmonic resonances. The authors included in the system model an inertial excitation force in addition to the eccentricity excitation. The analyses illustrated the occurrence of superharmonic resonances due to the interaction between crack breathing and the inertial excitation. Accordingly, they reported that the inertial excitation and crack depth significantly affect the superharmonic responses that can be utilized for crack diagnose purposes. Lu et al. [21] discussed nonlinear dynamics of a dual-rotor system with a breathing transverse crack in the hollow shaft of the high-pressure rotor. Saeed and Eissa [22] discussed the nonlinear vibrations of vertically supported Jeffcott system having a transverse crack. The authors obtained an analytical solution for the considered system. They concluded that the whirling oscillations at both subharmonic and superharmonic resonance cases are independent of the disc eccentricity and the angle between the crack and imbalance directions, while the main reason of their excitations is the exerted parametric force due to the shaft crack. Moreover, the high sensitivity of the subharmonic resonance to the crack presence makes it the optimal case to be utilized for the crack diagnosis in the rotating machinery. In most published articles that discussed the dynamical behaviors of the cracked rotor system, the authors studied a simple linear model simulating the dynamical characteristic of a horizontally supported Jeffcott rotor system. Moreover, they utilized different numerical methods to distinguish between the dynamics of the cracked and non-cracked rotor system.

In this work, the dynamical behavior of a horizontally supported nonlinear Jeffcott rotor system having a transverse crack in an analytical framework utilizing the multiple scales perturbation method has been studied. The system governing equations are derived based on the breathing crack model. Then, the amplitude–phase modulating equations governing the system lateral vibrations at primary resonance are extracted. The periodic-motion bifurcation diagrams are obtained in terms of the disc spinning speed, the disc eccentricity, and the crack depth. The influences of the angle between the crack and imbalance directions (γ) on the vibration amplitudes are explored. Then, the obtained bifurcation diagrams are verified numerically where a good agreement between the analytical and numerical results is achieved. Finally, the concluded remarks and a comparison with previously published articles are included.

2. System modeling and asymptotic analyses

Mathematical model governing the lateral oscillations in X_1 and X_2 directions of a horizontally supported nonlinear Jeffcott rotor that shown in Fig. 1a is given as follows [23,24]:

$$m\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 + k_2x_1^3 + k_2x_1x_2^2 = me_d\omega^2\cos(\omega t) \tag{1.1}$$

$$m\ddot{x}_2 + c_2\dot{x}_2 + k_1x_2 + k_2x_2^3 + k_2x_1^2x_2 = me_d\omega^2\sin(\omega t) - mg \tag{1.2}$$

where c_1 and c_2 represent the linear damping coefficients, k_1x_1 and k_1x_2 denote the shaft linear restoring forces, $k_2(x_1^3 + x_1x_2^2)$ and $k_2(x_2^3 + x_1^2x_2)$ are the shaft nonlinear restoring forces due to Hertz contact force and bearing clearance [23–25], e_d is the disc eccentricity, ω is the disc spinning speed, and mg is the disc weight. At static equilibrium, we have $\ddot{x}_1 = \ddot{x}_2 = \dot{x}_1 = \dot{x}_2 = \omega = 0$. Substituting this condition into Eqs. (1.1) and (1.2), we get from Eq. (1.2)

$$k_1x_{ss} + k_2x_{ss}^3 = -mg \tag{2}$$

where x_{ss} represents the static deflection of the shaft in X_2-direction due to the disc weight as shown in Fig. 1a. Therefore, at static equilibrium the geometric center of a horizontally supported Jeffcott rotor is $G(0, x_{ss})$. Now, let y_1, y_2 denote the dynamic deviations of G away