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Quasi-periodic aeroelastic response analysis of an airfoil with external store by incremental harmonic balance method

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ABSTRACT

This paper presents an investigation on the nonlinear aeroelastic system of an airfoil with external store by incremental harmonic balance (IHB) method. Besides solving limit cycle (LC) solutions, the IHB method is implemented to obtain quasi-periodic (QP) solutions by introducing multiple irreducible time scales. Steady state responses such as LC and QP oscillations obtained by the presented method are verified by numerical examples. One pair of Floquet multipliers for LC solutions first leave and then enter a unit cycle at complex conjugate values, indicating the existence of a secondary Hopf bifurcation and its reversal one. Along with the fundamental frequency of LC oscillation, an additional frequency arises at the secondary bifurcation, and finally disappears at the reversal bifurcation. The appearance and disappearance of the irreducible frequency cause the steady state responses changing from LC to QP and back to LC oscillation.

1. Introduction

In the past few decades, a huge amount of research has been reported on the nonlinear aeroelastic responses of airfoil [1–4]. One of the major categories of aeroelastic analysis is based on a typical model of rigid airfoil section [5–7], oscillating in pitch and plunge directions. Both theoretical analysis and wing tunnel tests show that, limit cycle oscillations (LCOs) can arise as the flow velocity increases beyond (or decrease below) a critical flutter speed, when there are nonlinearities such as cubic stiffness, piecewise freeplay and hysteresis [8–10].

A lot of contributions have been made to obtain and analyze LCOs of nonlinear aeroelastic systems. Numerical methods such as time marching algorithms include finite difference, Runge–Kutta (RK) and reduced order cyclic methods [11], etc. Analytical or semi-analytical approaches have stimulated the research interests of many researchers, possibly due to the limitation of numerical approaches. For example, the harmonic balance [12], homotopy analysis [13] and IHB methods [14,15] have been successfully implemented to obtain LC solutions.

Aside from LCOs, it has been reported that nonlinear aeroelastic systems of airfoil can also exhibit complex dynamic behaviors such as quasi-periodic (QP) [16] and chaotic responses [17]. Li et al. revealed that, time delay between the control input and actuator may cause high-frequency motions and QP vibrations [18]. Luongo and Zulli [19] found that QP responses can take place in NES-controlled system of an airfoil. By using the normal form theories and numerical approaches, Zhang and Chen [20] successfully predicted the existence of QP responses in the nonlinear aeroelastic system of an airfoil with an external store.

Currently, the analysis of QP response is usually carried out based on numerical integration methods [21]. There is a lack of analytical technique, which can be generally employed to solve and analyze QP responses. In fact, it has been a tough job for years to seek quasi-periodic solutions analytically [22,23]. In our experience, unexpected obstacles would be confronted inevitably even in the construction of semi-analytical solutions.

Semi-analytical approaches for QP solution quantification are usually based more or less on harmonic balancing technique. Guskov et al. [24] presented an HB-type method to study QP motions. Peletan et al. [25] constructed an approach named as QP-HB method for rotor–stator dynamic systems. Well-known, the incremental HB (IHB) method provides us with a convenient way to generate continuation of periodic solutions. Such method is also found to be applicable in solving QP solutions of nonlinear Jeffcott rotor system [26]. More recently, the IHB was modified by Huang et al. [27] by considering two time scales to study the quasi-periodic motions of an axially moving beam with internal resonance.

As for QP response analysis of aeroelastic systems, to the best of our knowledge there is much less published literature focusing on semi-analytical solution techniques. A mixed multiple scale/HB method was developed by Luongo and Zulli [19], to obtain the QP solutions of NES-controlled aeroelastic system. Mundis and Mavriplis [28] suggested

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Nomenclature

α	pitch angle of the airfoil about the mid-point, rad
β	pitch angle of the control surface about the hinge, rad
h	plunge displacement of the airfoil, m
h_1	non-dimensional plunge displacement, $h_1 = h/b$
t, t_1	non-dimensional time, real time (second), $t = Vt_1/b$
E	the elastic axis of wing
F	aerodynamic center
G	mass center
b	semi-chord length of the airfoil, m
ab	distance from the elastic axis to the mid-chord with a as a coefficient
$x_\alpha b$	distance from the airfoil mass center to the elastic axis
$x_\beta b$	distance from the mass center of the control surface to the mid-chord
$\bar{L}b$	distance from aerodynamic center to the mid-chord
cb	distance from the hinge of the control surface to the mid-chord
m	mass per unit length of airfoil
m_β, m_h	modal mass per unit span for each degree of freedom
m_t	total mass of the modal per unit span
$\omega_\alpha, \omega_\beta, \omega_h$	uncoupled natural frequency
r_α	radius of the gyration about the wing-aileron
r_β	reduced radius of gyration of aileron
K_α, K_β, K_h	the coefficients of linear stiffnesses of each degree-of-freedom
$\varsigma_\alpha, \varsigma_\beta, \varsigma_h$	the damping ratios of each degree of freedom
c_α, c_β, c_h	the coefficients of damping
$k_{\alpha 3}, k_{\beta 3}, k_{h3}$	the nonlinear stiffnesses of each degree of freedom
L	aerodynamic lift
M_L	aerodynamic lift moment
M_α	aerodynamic moment of wing-aileron
M_β	aerodynamic moment of aileron
S_α	the static unbalance moment of inertia about the elastic axis E
S_β	the static unbalance inertia moment of the external store about the aerodynamic center F
I_α	mass moments of inertia of the airfoil
I_β	mass moments of inertia of the store
V	flow speed, m/s
Q	non-dimensional flow velocity
Q_f	critical flow speed
ρ	air density $\mathrm{kg/m^3}$
μ	mass ratios of airfoil/air
μ_β	mass ratios of store/air

a hybrid approach, namely time spectral method, for solving quasi-periodic solution of aeroelastic system.

More and more QP responses have been found to take place in aeroelastic system [16–20], as mentioned above however, it is rare to find quantificative analysis via semi-analytical techniques. Different from the fact that there is a single fundamental frequency for LCOs [11–15], multiple irreducible frequencies are usually detected in QP responses. Both the fundamental and the additional irreducible frequencies have to be determined in the solution process for nonlinear aeroelastic systems, as these systems are self-excited without priorly given external excitations [19]. This feature makes it even tougher to search QP solutions of aeroelastic systems, when comparing to some other systems such as rotor–stator and Jeffcott rotor as they contain pre-established frequencies [25,26].

It is mentioned above, Zhang and Chen [20] revealed that there exist QP responses in the nonlinear aeroelastic system of an airfoil with an external store. The authors employed the normal form theory to examine

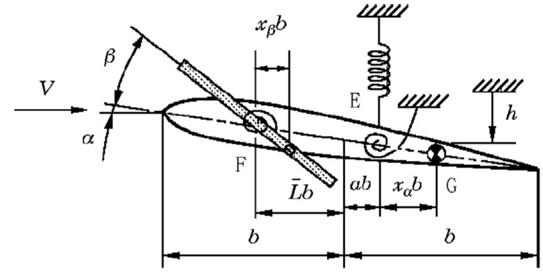

Fig. 1. Sketch of an airfoil with an external store [29].

the bifurcation of equilibrium qualitatively [20]. QP responses predicted by this method are verified by numerical solutions. Despite there are indeed several cases being reported for the existence of QP responses in aeroelastic systems, much fewer contribution has been made to understand the evolution of QP responses. The vibration evolution was investigated by Virgin et al. between LC, QP and chaotic responses for airfoil aeroelastic system with a freeplay [16].

In this paper, we will investigate the nonlinear aeroelastic system of an airfoil with an external store [20,29]. Different from the numerical computation of LCOs [29] and the qualitative investigation on equilibrium bifurcations [20], the presented study puts the major interest on accurate quantification of QP responses. The QP responses will be obtained semi-analytically by employing the IHB method with multiple time scales. Note also that, the existing studies consider either the plunge [20] or pitch nonlinear stiffness [29]; herein, we adopt store-associated nonlinearities such as a cubic stiffness in our considered system.

Furthermore, we are also curious about the evolution of LC to QP responses, as well as that of QP back to LC. Here we try to understand the phenomenological rather than mechanistic description of QP responses. Importantly, it is the semi-analytical feature of the IHB solution that provides us with a convenient way to analyze its evolution phenomenologically, as it is possible to generate solution continuation by the IHB method. Therefore, we will pay special attention on the harmonics associated with different time scales, in order to better understand the evolution of QP as well as LC oscillations in the considered system.

2. Equations of motion

The physical model shown in Fig. 1 is a two-dimensional airfoil with an external store. The airfoil section itself oscillates in the directions of pitch and plunge. The pitch angle about the elastic axis is denoted by α, positive with the nose up. The plunge deflection is denoted by h, positive in the downward direction. The external store is located at the aerodynamic center F with a distance from mid-chord as $\bar{L}b$. The varying pitch angle of the external store is denoted by β, positive with the nose up. The elastic axis is located at a distance ab from the mid-chord, and also at a distance $x_\alpha b$ from the mass center G. The mid-chord length is b and the mass center of the external store is $x_\beta b$, both measured from the aerodynamic center with positive values when measured toward the trailing edge of the airfoil.

Considering there is aerodynamic acting on the airfoil, while neglecting that acting on the external store, the coupled equations for the airfoil motion can be written as

$$\begin{cases} \left(m+m_\beta\right)\ddot{h} + \left(S_\alpha + S_\beta - m_\beta L\right)\ddot{\alpha} + S_\beta\ddot{\beta} + K_h h = -L \\ \left(S_\alpha + S_\beta - m_\beta L\right)\ddot{h} + \left(I_\alpha + I_\beta + m_\beta L^2 - 2S_\beta L\right)\ddot{\alpha} + \left(I_\beta - S_\beta L\right)\ddot{\beta} \\ \quad + K_\alpha \alpha = M_L \\ S_\alpha \ddot{h} + \left(I_\beta - S_\beta L\right)\ddot{\alpha} + I_\beta\ddot{\beta} + K_\beta \beta = 0 \end{cases} \tag{1}$$

where L is the lifting force and M_L is the lifting moment; $S_\alpha = mx_\alpha b$ is the mass static moment of the airfoil with respect to the elastic axis