ARTICLE IN PRESS

International Journal of Non-Linear Mechanics ■ (■■■■) ■■■–■■■

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Synchronized motion of noncontact rack-and-pinion devices subject to thermal noise

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ARTICLE INFO

MSC 2010:
00-01
99-00

Keywords:
Nanomachine
Lateral Casimir force
Thermal noise
Synchronization

ABSTRACT

We study a submicron device composed of one rack and N pinions. The pinions are coupled via the torsional springs. The rack and pinions have no contact, but are intermeshed via the lateral Casimir force. We show that even extremely soft torsional springs allow synchronized motion of N pinions. The total load that the machine lifts up, increases almost linearly with N. The synchronized state blooms even if the spring constants and the distances between the rack and the pinions are not tuned, and the thermal noise looms. These results lead one to be optimistic about harnessing the Casimir force at the nanoscale and the realization of a new generation of nanodevices.

1. Introduction

In 1873, van der Waals proposed the equation of state $(P+a/v^2)(v-b)=RT$ for the gases, where P, v, and T denote pressure, molar volume, and temperature, respectively. R is the universal gas constant, and a and b are constants appropriate to the gas. Van der Waals noticed that the constant a is associated with an interatomic force. This led to a quest for a universal interaction potential, independent of temperature and permanent dipole moments of atoms. In 1930, London derived an interaction potential $\propto 1/r^6$ between two atoms at a distance r. In fact London considered *zero-point energy* of the two atoms, modeled as harmonic oscillators, to explain the origin of van der Waals force. In his attempts to understand an interaction potential $\propto 1/r^7$ at large separations, Casimir attributed the van der Waals interaction to the change in the zero point energy of the electromagnetic field due to the presence of two atoms [1]. In 1948, Casimir predicted that two parallel conducting neutral plates immersed in the quantum vacuum, experience the force per unit area $F_{\text{normal}} = -\pi^2\hbar c/(240H^4)$, where H is the separation of the plates [2]. Since then, the fluctuation-induced interactions have received widespread attention in various areas [1,3–5], such as cosmology, soft condensed matter physics [6–8] and nanotechnology [9–14].

According to the recent advances in microelectromechanical systems (MEMS) [15], one can envision that the elements of future small devices are closer than 100 nm. In this regime, the Casimir force becomes significant. However, there are subtle questions concerning the actuation of nanomechanical systems by the Casimir force. (i) Is the Casimir force strong enough to affect the dynamics of a nanomechanical system? (ii) The Casimir force between two objects is a nonlinear function of their relative position and orientation. Thus Casimir machines are inherently *nonlinear* nanomechanical systems. Encouraged by the use of the nonlinear dynamics of MEMS to realize resonant mass sensors, inertial sensors and signal processing systems [16], can one exploit the nonlinear dynamics of Casimir systems? In recent years, some progress have been made in the area of Casimir mechanical systems. Experiments have confirmed the influence of the Casimir force on the operation of small devices. Chan et al. observed frequency shifts, hysteretic behavior and bistability induced by the nonlinear Casimir force in the frequency response of a periodically driven micromachined torsional oscillator [17,18]. To remedy the wear problem in nanomachines [19–21], it has been suggested that the quantum vacuum – which is much different from the nothingness – can be used to intermesh the *noncontact* parts of a device [22–26]. To avoid stiction and adhesion of surfaces, it is proposed to use apart corrugated surfaces and to employ the lateral component of the Casimir force to interlock them. The successful measurements of the lateral Casimir force [27–30], point out that the Casimir devices are realizable.

The simplest Casimir machine consists of one corrugated plate (rack) and one corrugated cylinder (pinion) that are kept at a distance from

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https://doi.org/10.1016/j.ijnonlinmec.2017.12.007
Received 18 August 2017; Received in revised form 7 December 2017; Accepted 8 December 2017
Available online xxxx

Please cite this article in press as: M. Miri, Z. Etesami, Synchronized motion of noncontact rack-and-pinion devices subject to thermal noise, International Journal of Non-Linear Mechanics (2017), https://doi.org/10.1016/j.ijnonlinmec.2017.12.007.

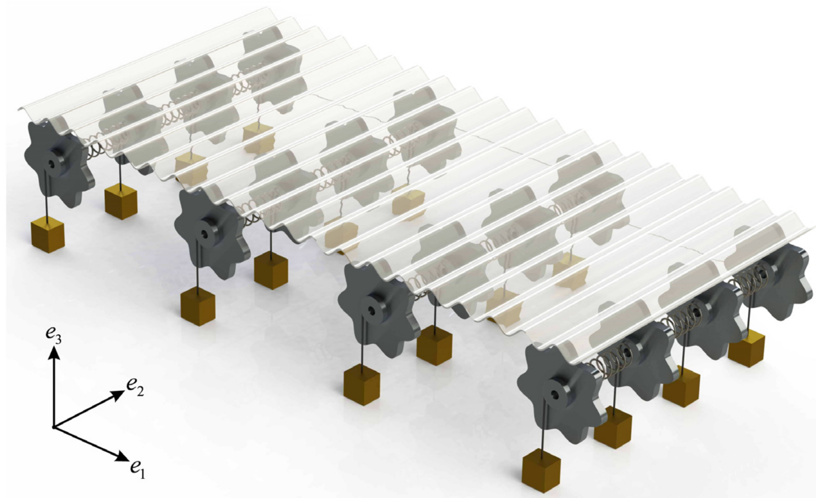

Fig. 1. The schematics of the Casimir device composed of one rack and $N = n \times n$ pinions arranged on a square array. Here $n = 4$. The adjacent vertical pinions (along the unit vector $\hat{e}_2$), and the last horizontal pinions (along the unit vector $\hat{e}_1$), are coupled via the torsional springs.

each other. The pinion experiences the lateral Casimir force [31,32]

$$F_{\text{lateral}} = -F \sin\left[\frac{2\pi}{\lambda}(x-y)\right], \quad (1)$$

where λ, x, and y denote the corrugation wavelength, pinion displacement, and rack displacement, respectively. $x = R\theta$ where R is the radius of the pinion and θ is the angle of rotation of the pinion. The Casimir force plays a key role in the equation of motion

$$\frac{I}{R}\frac{d^2x}{dt^2} = -RW - RF\sin\left[\frac{2\pi(x-y)}{\lambda}\right] - \frac{\zeta}{R}\frac{dx}{dt}, \quad (2)$$

where I is the moment of inertia of the pinion, ζ is the frictional coefficient, and W is the external load. The lateral Casimir force is not only a nonlinear but also an oscillating function of the displacement $x-y$. Thus even for the uniform motion of the rack, it is not immediately clear whether the time average angular velocity of the pinion is positive. In the weak dissipation regime, the Melnikov method has been used to study the phase portrait of the system [22]. In the strong dissipation regime, where the acceleration term is negligible, the equation of motion is solved analytically. It is found that for a large set of parameters, the device is able to lift up an external load W which is smaller than the amplitude of the lateral Casimir force F [22]. Typically $F \sim 1-10$ pN. It is further shown that the device is immune to the unavoidable thermal noise [23]. Note that the inertia of the miniaturized pinion is negligible. Moreover, the continuous bombardment of air (fluid) molecules that surround the axle of pinion, i.e. the thermal noise, markedly influences the dynamics of the pinion. However, in a large domain of parameter space and at room temperature, the device is able to lift up the load [23].

Arrays of *coupled* nonlinear mechanical systems exhibit a plethora of intriguing phenomena [16]. For example, intrinsic localized modes (ILMs) are reported to occur within microresonator arrays [33,34]: As a result of the nonlinearity of the system rather than the presence of an impurity or defect, the energy becomes localized at a certain location in the array. ILMs can be realized as nonlinear vibration modes [35]. Furthermore, it is shown that the lone white noise excitation does not produce ILMs in an array of coupled nonlinear oscillators. However, a combination of deterministic forcing and a noise strength higher than a threshold value, attenuates localization at one location whilst sustains it at another location in the array [36]. Another noteworthy phenomenon which arises in coupled resonator arrays is synchronization. Since Christiaan Huygens, who in 1665 observed and explained the synchronization of two pendulums placed near each other, many progresses have been made [37]. For example, oscillatory neurocomputer having autocorrelative associative memory based on a network of MEMS oscillators [38], noise-induced phase synchronization in chaotic oscillators [39], synchronization of micromechanical oscillator arrays coupled through light [40], and nonlinearity-induced synchronization enhancement in micromechanical oscillators [41] are reported. Inspired by these studies of coupled nonlinear mechanical systems, a Casimir device composed of one rack and *two* coupled pinions is designed [24]. It is shown that a coupling between two pinions via a torsional spring allows *synchronized motion* of two pinions and the lift of a total load larger than F. However, the impact of thermal noise on the synchronized state of the machine is not studied.

A Casimir device which manages a large external load $\sim$ 1000 pN is of immediate interest. It is logical to consider a device composed of one rack and N coupled pinions, see Fig. 1. The pinions are coupled via the torsional springs. Each pinion is subject to an external load W. In order to judge whether this machine is promising or not, the following questions must be answered: (i) Does the synchronized state exist for large values of N? (ii) Does the total load that the machine lifts up, increase sufficiently fast as N increases? In other words, is the system scalable? (iii) In practice, neither all gap sizes nor all torsional stiffnesses can be tuned to a particular value. Does the synchronized state exist in the simultaneous presence of the thermal noise and the structural randomness? (iv) Does the synchronized state bloom for moderate values of torsional stiffness? We find that the answer to all these questions is in the affirmative. It is of immense practical importance that the synchronized state of the Casimir device is quite robust. Indeed this leads one to be optimistic about harnessing the Casimir force at the nanoscale and the realization of a new generation of nanodevices.

The rest of the paper is organized as follows. Section 2 presents the set of Langevin equations describing the dynamics of pinions. Our results on the performance of the Casimir device are in Section 3. Section 4 is devoted to the typical values of parameters of the device. Remarks and conclusion are in Section 5.

2. Model

The Casimir device shown schematically in Fig. 1, is composed of one corrugated plate (rack) and N corrugated cylinders (pinions). We denote the corrugation amplitude, gap size, length, outer radius, inner radius, angle of rotation, lateral displacement, angular velocity, and velocity of the ith pinion by a_i, H_i, L, R, $r \approx R$, θ_i, $x_i = R\theta_i$, $d\theta_i/dt$, and $dx_i/dt = Rd\theta_i/dt$, respectively. The corrugation amplitude and lateral displacement of the rack are a_r and y, respectively. See Fig. 2(a)–(c). The wavelength of corrugation on the rack and all the pinions is the same λ to ensure coupling via the electromagnetic zero-point field. The Casimir device is in contact with a heat bath at temperature T_B. Each pinion is subject to an external load W.

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