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Analytical solutions of a nonlinear two degrees of freedom model of a human middle ear with SMA prosthesis

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ABSTRACT

A nonlinear two degrees of freedom system with shape memory element (SMA) is presented in the paper. Thermomechanical properties, shape memory effect and pseudoelasticity of the SMA joint, are modelled by phenomenological approach with the help of a fifth order polynomial approximation. The analytical solution of the mathematical model is obtained by means of the multiple time scales method. Next, a numerical study of a biomechanical system of human middle ear is performed. It has been demonstrated that for a selected rod type prothesis the cubic nonlinearity has the most essential effect on the system dynamics in the first order approximation, which converges with direct numerical simulations.

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1. Introduction

Shape memory alloys (SMAs) are attractive in various science and engineering applications, from biomedical to aerospace industry. They are used to built thermal actuators, stabilising mechanisms and so on. SMAs are a family of metals with the ability to change shape depending on their temperature [1]. SMA elements may exhibit one-way or two-way shape memory effect. The main difference between these effects is that no reverse change occurs after cooling in the case of one-way shape memory effect, whereas for two-way effect a change in shape during cooling occurs.

SMAs undergo thermo-elastic phase transformations between martensite and austenite, which can be induced by temperature or stress. The martensitic phase can have two variants: twinned or detwinned. The shape memory effect occurs at temperatures that are below a critical value, where twinned martensite is stable when free from stress. The conversion from twinned to detwinned martensite takes place by means of loading process. When the loading–unloading process is finished, some amount of residual strain remains, meaning that the reverse transformation from detwinned to twinned martensite is not completed. The shape memory effect appears by heating which activates the transformation from detwinned martensite to austenite. At temperatures that are above another critical value, when a piece of SMA is stressed at the constant temperature, inelastic deformation is observed above a critical stress. However, this inelastic process may fully recover during the subsequent loading. The stress–strain curve is the macroscopic manifestation of the deformation mechanism of the martensite. The curve forms a hysteresis loop known as pseudoelastic effect [2].

These thermomechanical properties of SMAs can be studied by microscopic or macroscopic models. The microscopic models treat phenomena on a molecular level. The macroscopic phenomenological models are more attractive from practical point of view. In the literature one can find different functionals representing the free energy and pseudo-potential of dissipation [3]. There are models (a) with polynomial free-energy, (b) with assumed transformation kinetics, (c) with internal restrictions.

The model with polynomial free-energy is based on Devonshire theory for temperature induced phase transition, combined with hysteresis which was proposed by Falk [4] and Falk and Konopka [5]. In that model free energy depends on the temperature and the one-dimensional strain without internal restrictions. The model is investigated here and therefore, more details are presented in the next section.

The model with assumed transformation kinetics, developed by Tanaka and Nagaki [6], additionally considers a scalar internal variable which characterises the extent of martensitic transformation. The paper provided the motivation for other researchers to propose modified transformation kinetics laws, e.g. Liang and Rogers [7] and Brinson [8]. Those models are the most popular in the literature, and play an important role in modelling of SMA actuators influenced by temperature [9].

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The model with internal restrictions was presented by Frémond [10], who developed a model which can reproduce the pseudoelastic behaviour and shape memory effect using three internal variables representing the volumetric fractions of two variants of martensite (twinned and detwinned) and the austenite parent phase. However, simplified one-dimensional models are often built upon the original concept of Frémond [11,3,12,2]. These models are simpler to analyse and give quite good approximation. Better and more reliable—thermomechanically-based models are presented for instant in [13,14].

Shape memory oscillators are popular in many scientific considerations and engineering applications. One and two degrees of freedom oscillators with an element (spring, rod, beam) made of SMA are analysed under different angles. Savi and Pacheco [1] analysed numerically free and forced vibration of 1 and 2dof SMA oscillator paying the main attention to on chaotic response. The more detailed analysis of 2dof system is presented in [15] and 1dof system in [16–19]. The authors investigate some aspects of bifurcation and hysteresis phenomenon. 1dof SMA oscillator is quite thoroughly investigated analytically by means of the multiple time scales method in [20,21] and also experimentally in [22,23].

From engineering point of view, SMA systems can be applied to control or avoid instability and chaos. For instance, in the paper [24] a NiTi SMA micro-valve design for passive flow control and thermal regulation is prototyped at a macro scale. Optimal linear control in SMA nonlinear oscillator is discussed in [25]. A time delay procedure to control chaos in SMA system is presented in [26]. A control problem of SMA composite wings is raised in [27], whereas a control of a rotordynamic system is presented in [28]. To identify dynamical behaviour of SMA systems, new procedures are proposed such as: 0–1 test [29] and wavelet transform [30].

In this paper we promote an idea of a human middle ear prosthesis made of SMA. When the ossicular chain structure is destroyed by inflammatory diseases such as chronic suppurative otitis media or cholesteatoma, the middle ear should be reconstructed. In the medical practice it is called ossiculoplasty (the reconstruction of the middle ear ossicles). For over 50 years, middle ear surgery techniques have improved hearing that had been destroyed by various diseases. Numerous procedures are currently used in clinical practice, and a variety of middle ear prostheses made of titanium or titanium alloy are available. Most prostheses can be modified only once they have been prepared. If a prosthesis is too short, it cannot be remodelled, it must be lost. However, prostheses of different sizes can be chosen or tailored by a surgeon to the desired length. Their length can be adjusted by cutting the prosthesis leg. The prosthesis can be implemented in a damaged middle ear to connect the malleus (and the tympanic membrane) with the stapes in order to repair sound transmission process from the outer to inner ear. The problem of middle ear reconstruction is described in literature but usually classical prostheses made of titanium are used and tested numerically [31,32] and experimentally [33–37]. The presented prostheses cannot be modified after they have been prepared (cut) by surgeon during a medical operation. Therefore, here a new idea of the prosthesis made of Nitinol or Flexinol which can change its length, and first of all, can adjust its size and angulation to the requirements of a particular medical case is presented. Different shapes of the SMA prostheses are presented in [38,39]. The prosthesis should improve and fasten implementation process. The SMA should have a one-way shape memory effect because it must change its shape while being heated and remain unchanged after cooling. During an operation, point-heating can be realised with a bipolar coagulation or a laser beam (CO_2 or diode).

The main aim of the paper has two aspects. Firstly, we would like to get analytical solutions of periodic motion of two degrees of freedom system with SMA joint to compare these results with numerical outcomes reported in [40], and secondly to extend the numerical findings by analytical formulae. In this paper, the same polynomial model of SMA element is applied as in [40]. The SMA element is placed into the nonlinear two degrees of freedom system. Then the

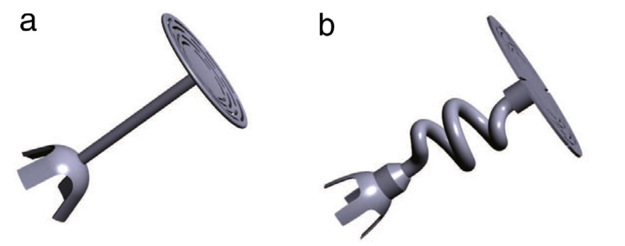

Fig. 1. Examples of shape memory prosthesis, (a) stiff of a rod type prothesis and (b) flexible of a spring type prothesis.

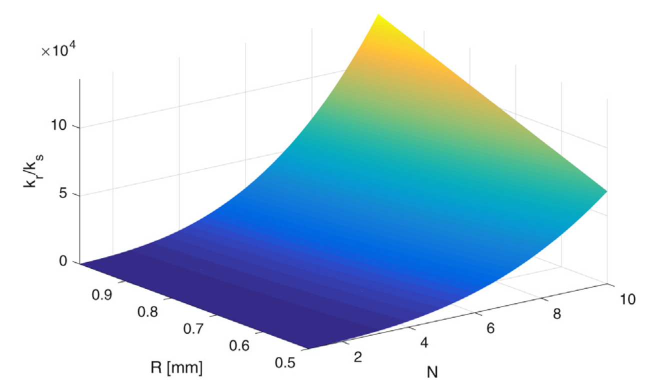

Fig. 2. Stiffness ratio of the rod to the spring prothesis against radius of a spring R and a number of spring coils N.

analytical solution is obtained by means of the multiple time scales method. The solution is general and can be applied to any mechanical problem. However, a detailed numerical study is performed for of a biomechanical 2dof system of a human middle ear reconstructed by means of SMA prosthesis. The reconstructed joint can be designed considering a wide range of structural elements, having different shapes, masses and stiffnesses. Examples of the SMA prothesis (the SMA joint) are presented in Fig. 1. Fig. 1a shows the prothesis as a simple SMA rod with special structural endings which enable its comfortable fixing during surgery. The stiffness of this prothesis type depends on the rod diameter and length, but in general it is relatively stiff element. The second option (Fig. 1b) is the prothesis of a spring shape, giving possibility to fit its properties to a whole ossicles chain, in order to get a required sound transmission for specific frequencies. The spring type prothesis enables a change of its stiffness in a very wide range by varying spring radius R and a number of coils N (Fig. 2). The demonstrated k_r/k_s factor means the stiffness ratio between the rod (Fig. 1a) and the spring prosthesis (Fig. 1b).

Often, in the clinical practice the prothesis is a just stiff rod made of titanium which just connects the malleus with the stapes. Thus, in this paper we take data for the stiff rod type prothesis, presented in Fig. 1(a). Nevertheless, the obtained analytical solutions will be used in future analysis for a design of a spring type prothesis which can be fitted to the patient individual properties and the middle ear chain. SMA is employed to built the prosthesis to give a possibility of length control activated by heating. Providing a proper portion of thermal energy, during implementation, the prosthesis can be heated locally above the temperature of activation (temperature of austenite phase). Then, the prosthesis get lengthen in controllable way. To achieve the proper length, heating should be repeated several times.

2. Model of a system with a shape memory element

We start the study from a general model with two degrees of freedom which consists of two lumped masses, springs and dampers (Fig. 3).

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