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Instability investigation for rotating thin spherical membrane

Yang Zhou [1], Arne Nordmark [1], Anders Eriksson [*,2]

KTH Mechanics, Royal Institute of Technology, Osquars backe 18, SE-100 44 Stockholm, Sweden

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ABSTRACT

A fluid-filled truncated spherical membrane fixed along its truncated edge to a horizontal, rigid and frictionless plane and spinning around a center axis was investigated. A two-parameter Mooney–Rivlin model was used to describe the material of the membrane. The truncated sphere was modeled in 3D using finite element meshes with different symmetry properties. A quadratic function was used for interpolating hydro-static pressure, giving a symmetric tangent stiffness matrix, thereby reproducing the conservative problem. Various problem settings were considered, related to the spinning, and different instability behaviors were observed. Multi-parametric problems were defined, generalized paths including primary and secondary paths were followed. Stability of the multi-parametric problem was evaluated using generalized eigenvalue analysis based on the total differential matrix for the constrained problem. Numerical results showed that mesh symmetry affected the simulated stability behavior. Fold line evaluations showed the parametric effects on critical solutions.

1. Introduction

Utilizing their small thickness and low weight, thin membranes have been used in many fields. Gas-inflated balloons are commonly used in angioplasty, [1,2], architecture [3,4] and space engineering, [5,6]. Fluid-pressurized membranes are also the containers for most of the biological organisms, [7]. Gas-inflated membranes are widely investigated in the literature, [8,9], showing the large deformations of thin membranes and their proneness to lose stability with respect to gas pressure. As a deformation-dependent loading, one way to introduce the fluid pressure in finite element context is developed in [10] considering the effect on stability. Numerical simulations and some experiments performed on fluid pressurized membranes are discussed in [11], showing comparisons between different material models. The equilibrium configurations for an indented fluid-filled spherical membrane are analyzed in [12], showing the relation between principal stress and sharpness of the indenter. A spherical membrane filled with fluid and in contact with two parallel horizontal rigid planes is analyzed in [13], where the wrinkling effect is also discussed. A spherical inflatable membrane under hydro-static load is analyzed in [14], and many other interesting settings have also been investigated.

Large deformations as well as stability are often studied for thin membranes under various loading conditions, exhibiting the mechanical response. The instabilities occurring for membranes can be classified as global instabilities, e.g., limit points or bifurcation points, and local instabilities, e.g., wrinkling. An eigenmode injection method was proposed as a method to initiate the secondary paths at the bifurcation points, [15], and used in [16] to track the secondary paths of two connected air-inflated balloons. A perturbation method has been used to evaluate the stability of an air-inflated balloon, and different conclusions were obtained with respect to gas pressure and mass, [17]. This conclusion, as well as those from other studies [18–20], underlines the importance of specifying the variables for interpreting the stability, especially for multi-parametric problems. The sufficient condition for static stability based on the eigenvalues of the tangent stiffness matrix has been commonly used in finite element context, [21,22]. Based on the static stability criterion, multi-parametric stability of thin membranes with physical constraints were discussed in [23,24].

Well-known contributions from [25–27] propose a relaxed strain energy function for a wrinkling case, instead of the standard strain energy density function for a hyper-elastic material. However, recent work has shown that with pressure loading, the relaxed energy function may not guarantee a stable configuration within the wrinkled region, [28].

The membranes under discussion usually have highly symmetrical geometries, e.g., the circular membranes subject to gas and fluid pressure [29–31], the spherical membrane containing gas and fluid [18,32],

* Corresponding author.
E-mail address: anderi@kth.se (A. Eriksson).
[1] Ph.D.
[2] Professor.

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the cylindrical tube under axial load, [33], and the square membrane inflated by gas [34,35]. Group-theoretic methods have shown potential to problems which exhibit symmetry, as they can reduce the computational efforts, [36]. For structures with high symmetry, sub-domains of the structures have been commonly used, instead of the whole domain for investigations, but previous works showed the effects of the symmetries on computational results [18,37]. In particular these show how results concerning bifurcations, which can be seen as a symmetry-breaking response for symmetric structures, are affected by the modeling.

In the present work, a fluid-filled truncated sphere fixed to a plane and spinning around the center axis was investigated. Even if rotating pressurized membranes have some applications, the main motivation for the study was an investigation of the interaction between two distinct load effects, each one well-known to lead to instabilities. The problem was only considered as quasi-static, and the problem setting considered the spinning velocity and the angular momentum. Different instability behaviors were observed, related to the used meshes. Hence, the effects of the discretization on the instability behaviors were investigated. Loading derived from the hydro-static pressure was implemented analytically. The reasons for this fully accurate implementation are given, and related to the symmetry of the tangent stiffness matrix. Generalized equilibrium paths were followed, and stability responses for multi-parametric problems with augmenting physical constraint equations are reported, showing interactions between parameters in instability.

2. Mathematical models

2.1. Problem description

In this paper, we consider a truncated spherical membrane, which is fixed with its truncated edge to a ring, which is in turn fixed to an infinite x–y plane, Fig. 1. In the treatment, the membrane was assumed as massless, and to be completely filled with fluid. We considered it as spinning around the z-axis together with the rigid plane, but also the fluid enclosed, without any relative movements between membrane and fluid. Except for the fixed truncation edge, contacts between any part of the membrane and the rigid surface were seen as hard and frictionless, but modeled through a penalty formulation for normal penetration. Under these assumptions, the problem was treated as quasi-static, with gravity and angular momentum considered as loadings.

The un-deformed radius, uniform thickness and truncated height below the equator were denoted as R_o, t and h_o, respectively, Fig. 1(a). The truncated sphere was modeled in 3D, so that non-axisymmetric response, in particular post-buckling behaviors, could be observed. The fluid pressure p_f was evaluated as the sum of contributions from gravity and centrifugal forces. Gravity thereby gives a pressure which is linearly distributed in the gravitational direction, depending on a fluid surface level, with z coordinate H, which would be the free fluid surface level at the axis of rotation. The centrifugal force, from the heavy fluid only, gives a pressure quadratically varying from the z-axis to the sphere surface, depending on the angular velocity ω around the z axis. In addition to the most conveniently used loading parameters H, ω, fluid volume V_f and angular momentum L were also considered. The possible variables and quantities are shown in Fig. 1.

2.1.1. Membrane material

An isotropic, incompressible and hyper-elastic material was used to describe the membrane. The constitutive relation between stress and strain was given from a strain energy density $\overline{W}_s$ as [38]:

$$\boldsymbol{S}(\boldsymbol{C}) = \lambda \boldsymbol{C}^{-1} + 2\frac{\partial \overline{W}_s}{\partial \boldsymbol{C}}, \tag{1}$$

where λ is a Lagrange multiplier enforcing incompressibility. In the equation, $\boldsymbol{S}$ was defined as the second Piola–Kirchhoff stress tensor:

$$\boldsymbol{S} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{21} & S_{22} & 0 \\ 0 & 0 & S_{33} \end{bmatrix}, \tag{2}$$

with $S_{33} = 0$ due to the local plane-stress assumption. The strain $\boldsymbol{C}$ was defined as the right Cauchy–Green strain tensor:

$$\boldsymbol{C} = \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{21} & C_{22} & 0 \\ 0 & 0 & C_{33} \end{bmatrix}, \tag{3}$$

with the initial configuration considered as the reference state, and C_{33} defined from the other components due to the assumption of incompressibility.

A two-parameter Mooney–Rivlin material model was used to describe the strain energy density:

$$\overline{W}_s = c_1(I_1(\boldsymbol{C}) - 3) + c_2(I_2(\boldsymbol{C}) - 3), \tag{4}$$

where I_1 and I_2 are the first two invariants of the strain tensor $\boldsymbol{C}$, and the two parameters c_1, c_2, restricted as:

$$\frac{\mu}{2} = c_1 + c_2 \tag{5}$$

$$k = \frac{c_2}{c_1}, \tag{6}$$

in which μ and k are constitutive constants, often seen as a shear modulus, and a 'hardening factor', respectively.

2.1.2. Rigid contacts

In addition to the fixation of the truncation edge, a frictionless hard contact between the truncated sphere and the horizontal plane was defined point-wise, using the contact pressure p_c and the distance δ between them, Fig. 1:

$$\begin{cases} p_c \delta = 0 \\ p_c \geq 0 \\ \delta \geq 0 \end{cases} \tag{7}$$

Clearly, contact pressure p_c vanishes when the membrane is not in contact, $\delta > 0$, while δ must vanish for the pressure to be active, $p_c > 0$. As discussed below, the hard contact was implemented as a penalized contact.

2.1.3. Fluid pressure evaluation

A fluid-filled membrane was investigated in the current work, cf. [23]. The fluid pressure applied on the membrane surface was evaluated separately as a linearly varying pressure p_g, from gravity, dependent on the current coordinate z and the gravitational zero over-pressure level H. A quadratically varying pressure p_s, due to centrifugal force was added, dependent on the current coordinates x, y and the angular velocity around the z axis ω. With the density of fluid ρ and the gravity acceleration g, the total pressure was seen as:

$$p_f(x, y, z) = p_g + p_s = \rho g(H - z) + \frac{1}{2}\rho\omega^2(x^2 + y^2), \tag{8}$$

with the total pressure acting in a normal outwards direction to the deformed membrane.

In the evaluation for the fluid pressure, Eq. (8), it was assumed that p_g, p_s were independent, considering H, ω as the primary loading parameters, respectively. More practical terms, e.g., the fluid volume V_f, [23], and the z component of angular momentum,

$$L_f = \omega \int_{V_f} \rho(x^2 + y^2)\, dV_f, \tag{9}$$

which is integrated over the deformed volume, are also considered in the discussion, with more details given below.

2.2. Modeling

The truncated spherical membrane discussed here is axi-symmetric in its continuous modeling, i.e., has $C_{\infty v}$ symmetry, [39]. For this, the typical eigensolutions can be classified into three subsets:

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