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Control of localized non-linear strain waves in complex crystalline lattices

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ABSTRACT

The distributed feedback control is developed to support propagation of localized non-linear waves for the double sine-Gordon equation and the dispersive sine-Gordon equation previously obtained for the description of dynamic processes in complex crystalline lattices. The control allows the propagation of both bell-shaped and kink-shaped waves with permanent shape and velocity, with a functional form which does not correspond to the known analytical solutions of the equations. The results might be used for choosing external loading providing desired strain localization or variations in the internal structure of the lattice.

1. Introduction

Continuum approximation is an efficient method for studying dynamical behavior of strains in lattices [1]. More complex lattice structures may be found than the usual lattice with equal masses whose interaction is modeled by central springs [2–8]. In particular, one can consider an external action on the lattice such as the Frenkel–Kontorova model [3]. An additional rotational degree of freedom may be taken into account in the discrete chain model [1,4,5]. The di-atomic model involves masses of two kinds in the lattice [6,7]. Non-neighboring interactions also make standard lattice model more complex [8].

The inter-mass interactions in the lattice may be weakly non-linear or strongly non-linear. In the former case, non-linear terms in the governing equations have power series form. In the latter case, the sine-Gordon equation appears in the Frenkel–Kontorova model [3,9]. Also the double sine-Gordon equation is obtained for a compressible chain of dipoles in Ref. [5], and a more general model is developed in [4] but its solution relates to that of the double sine-Gordon equation. Cardinal variations in di-atomic lattice were considered in Refs. [10,11]. They result in the description of internal variations of the structure of the lattice, moving defects, etc., and to the development of the highly non-linear model functionally similar to that studied in [4]. Strongly non-linear description with non-neighboring interactions results in obtaining the sine-Gordon equation with higher-order dispersion terms or the dispersive sine-Gordon equation [12,13].

Localized waves of stationary shape arise as solutions of many non-linear wave equations. However, the form of the solution may be varied since these solutions are supported by a balance between non-linearity and other features of the equation, first of all, dispersion. The balance may provide different forms of the wave: some equations possess bell-shaped localized wave solutions, others kink-shaped solutions. Of course, the initial conditions should be suitable for stable localized wave propagation. Localized waves play an important role in various physical problems. They transfer considerable energy, and sometimes may result in damage of a waveguide or onset of a defect in a crystalline lattice. At the same time, waves of stationary shape and velocity transmit information about the physical properties of a medium where they propagate since their amplitude and velocity depend upon the characteristics of the medium. Therefore it is important to know how to support the propagation of such localized waves and make their existence less sensitive to the initial perturbations. One possibility to achieve the desired shape and velocity of the wave is by application of the methods of control [14,15].

Development of the control mechanisms in non-linear wave processes usually relies on asymptotic methods [16–20]. One can

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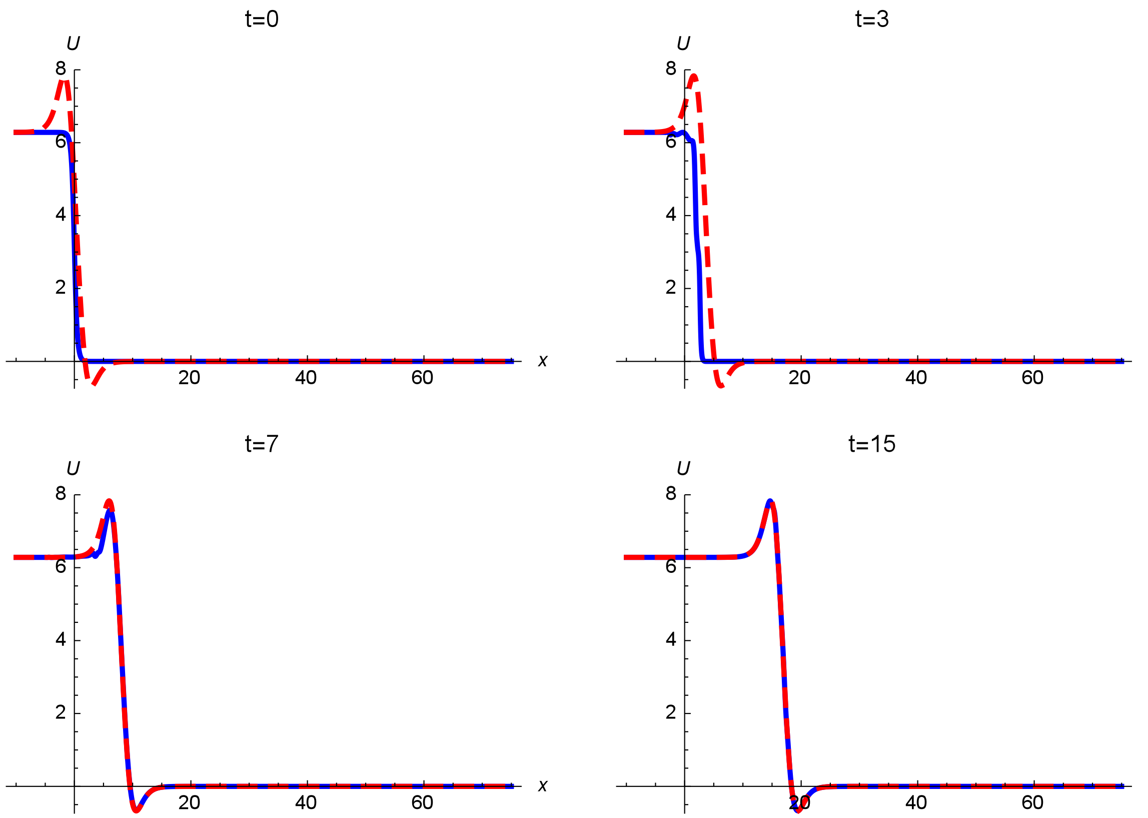

Fig. 1. Generation of the kink-shaped wave for Eq. (6) at $q = 10$. The dashed line is the desired traveling wave (13).

control the boundary conditions to achieve significant difference in the wave behavior [21,22]. However, this looks problematic for a control of localized waves. Another way is to control the value of one of the coefficients of the equations [14,20]. An additional control term may be added to the equation [23]. However, the addition of the term should be justified, in particular, using the speed-gradient control approach [14].

Distributed control means that the control function is not spatially uniform like in many vibration control problems. One call control *non-distributed* if the control function is a function of time only. The expression of the control function may include dependence of the state (potentially measured) variables. In this case it is called feedback control. Otherwise, one can call the control *non-feedback control*. Distributed control systems were studied in the literature since the 1960s [24–26]. The interest in this area was mainly theoretical though most real world controlled systems are spatially distributed and described by partial differential equations. In most existing results the control goals are either stabilization of the fixed system state (regulation) or tracking of the desired state as a function of time.

Here, the control function is added to the equation under study. To obtain the resulting expression for it, fulfillment of some reasonable energetic equalities is used, e.g., achievement of the prespecified level of energy. In particular, this condition may be obtained from conservation of the Hamiltonian for the equation under study. A non-distributed control function may be obtained in this way. Otherwise, satisfaction of a functional in non-integral form may be used, if the minimum of the functional corresponds to the achievement of the goal of the control, e.g., achievement of the wave with desired shape and velocity. In our paper a more efficient approach to finding the control function based on the speed-gradient method is used. The differentiation of the functional along the controlled system trajectories is performed in order to make it explicitly dependent on the control function.

Up to now, it seems that one cannot explain analytically why the control works for the wave equations. In previous papers [27–29] we checked various kinds of the control function using both feedback and non-feedback control [28]. We also used both distributed and non-distributed controls [27]. Now one can definitely say that non-distributed control does not provide localization of the wave. Non-feedback control is not efficient and not universal for various kinds of desired wave profiles. Moreover, the control function should include tendency both to the desired shape and to the desired velocity of the wave. In previous studies for ordinary differential equations the idea of the speed-gradient method was to achieve a decrease of some goal function along trajectories of the closed loop system. It means that the goal function may play the role of a Lyapunov function, which makes stability of the control system plausible. Although this idea may be extended to partial differential equations, such an analysis is beyond the scope of this paper.

In previous papers [27–29] it was found that distributed control may provide localization of the wave. However, the desired shape and velocity of the waves considered in Refs. [27–29] corresponded to known analytical solutions to the sine-Gordon equation. In this paper, we extend the distributed feedback algorithm of control to support stable propagation of localized non-linear waves which do not relate to any analytical solution. Two equations will be studied. The first of them is the double sine-Gordon equation, and another equation is the dispersive sine-Gordon equation. The paper is organized as follows. First, some known localized traveling wave solutions that require specific initial conditions and restrictions on the equation coefficients for existence are recalled in Section 2. Then the distributed feedback control algorithm is developed in Section 3 to achieve the desired stable propagation of localized wave of stationary shape and velocity. This algorithm is used in Sections 4–6 to obtain numerically stable propagation of kink-shaped and bell-shaped solitary waves