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A geometrically non-linear plate model including surface stress effect for the pull-in instability analysis of rectangular nanoplates under hydrostatic and electrostatic actuations

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ABSTRACT

Presented herein is a comprehensive investigation on the size-dependent pull-in instability of geometrically non-linear rectangular nanoplates including surface stress effects undergoing hydrostatic and electrostatic actuations. To this end, based on the Gurtin–Murdoch theory, a non-classical continuum plate model capable of incorporating size-effects is developed; then, by means of the principle of virtual work, the governing equations of the actuated nanoplate are obtained. Subsequently, the generalized differential quadrature (GDQ) method is used to discretize the governing equations and associated boundary conditions, before solving numerically by the pseudo arc-length algorithm. Finally, the influences of important parameters including the geometrical non-linearity, thickness of the nanoplate, surface elastic modulus, residual surface stress and boundary conditions on the pull-in behavior of the actuated nanoplate are thoroughly studied. In addition, the effect of the material on the pull-in voltage and pressure is investigated by comparing the results obtained from nanoplates made of two different materials including aluminum (Al) and silicon (Si).

1. Introduction

List of endless applications of micro and nanoplates in the areas of micro and nano-electromechanical systems (MEMS and NEMS) has motivated scientists to broaden their investigation to comprehend all aspects and phenomena associated with these classes of new materials. The pull-in instability is a discontinuity related to the interaction of the elastic and the electrostatic forces, and was firstly introduced by Nathanson et al. [1] and Taylor [2]. By imposing a potential difference, the structure deforms due to electrostatic forces. When the applied potential difference surpasses a critical value which is popular as pull-in voltage, the elastic force can no more withstand the electrostatic force and the system fails to establish a force balance without a physical contact; hence, collapse would be unavoidable. The pull-in instability is mostly studied in the framework of classical continuum theories; while, these theories are not capable of considering size effects in nanostructures; conversely, it is demonstrably approved that the pull-in instability of nanomaterials are size-dependent [3]. Accordingly, a primacy must be given to developing unconventional continuum theories able to incorporate size effects in micro and nanostructures such as non-local elasticity, couple stress elasticity, strain gradient elasticity and surface elasticity theories [4–11]. One of the most effective size dependent theories is proposed by Gurtin and Murdoch [10,11] in which they developed a theoretical concept based on the continuum mechanics including surface stress effects. From their proposition, the surface layer of a solid is a mathematical layer of zero thickness with different material properties from the underlying bulk that is completely attached by the membrane. This theory has been applied by many researchers to study the mechanical behavior of nanostructures [12–15]. Based on a size-dependent model, Miller and Shenoy [16] studied the pure bending and unidirectional tension of nanobars and nanoplates. Their results were indicated to be in excellent agreement with the atomistic simulation results via the selection of suitable material constants for the surface layer. Moreover, based upon the Gurtin–Murdoch elasticity theory, Shenoy [17] investigated the size-dependent torsional rigidities of nanosized structural elements with considering surface energy effects. Sapsathiarn et al. [18] proposed a surface stress beam model using the Gurtin–Murdoch elasticity theory and showed that their model can predict the experimental results through size-independent properties such as bulk modulus and surface residual stress.

In MEMS and NEMS, when the dimension decreases, the surface forces such as Casimir force, van der Waals and electrostatic forces become more important. In a realistic case, the Casimir interaction

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between two surfaces largely depends on the several factors such as the dielectric properties of the surfaces and the geometric parameters [19,20]. According to the equilibrium conditions, in the absence of external loading, the surface tension initiates a compressive residual stress field in the bulk of the nanoplate. Hence, based on generalized Young–Laplace equations, it is deduced that the surface tension reveals itself in a non-classical boundary condition giving the force in the bulk of nanostructures to equilibrate the surface tension. The stress field in the bulk and residual surface stress are considered as a residual stress field in nanostructures, which are not generally homogeneous and are associated with zero traction on the boundary of nanostructures. Since the nanoplate will be very thin, the residual stress is high. Residual stress in the bulk can be computed by the equilibrium condition. This self-equilibrium condition (without external loadings) under the action of surface tension is usually recognized as the reference configuration, from which nanostructures experience an elastic deformation. There are some rare works in the literature which have recognized the significance of surface tension and the residual stress field in the bulk induced by surface tension on the elastic deformations of nanostructures [10,13].

Only a small proportion of investigations is devoted to study the size-dependent pull-in instability of nanostructures. In this direction, Fu and Zhang [21] represented a modified continuum model of electrically actuated nanobeams by considering surface elasticity. They simulated the surface layer by means of Gurtin and Murdoch's theory of surface elasticity and modeled the bulk deformation kinematics by employing the Euler–Bernoulli beam theory. Furthermore, in a recent work, Ansari et al. [22] on the basis of a modified continuum model investigated the size-dependent pull-in behavior of hydrostatically and electrostatically actuated rectangular nanoplates including surface stress effects. They used the Gurtin–Murdoch theory and Hamilton's principle to obtain the governing equations.

To our best knowledge, all efforts concerned with the pull-in phenomenon are restricted to linear studies; this paper is accomplished to shed light on the pull-in behavior of geometrically non-linear rectangular nanoplates including surface stress effects and undergoing hydrostatic and electrostatic actuation. To this end, the Gurtin–Murdoch theory is employed to consider both size and surface stress effects. Based on the principle of virtual work, the governing equations and associated boundary conditions are derived and then after being discretized by the GDQ method, are solved numerically through pseudo arc-length algorithm. Finally, the results obtained from linear and non-linear responses are compared and effects of the thickness of nanoplates, surface elastic modulus, residual surface stress and boundary conditions on the pull-in voltage and hydrostatic pressure of the actuated nanoplate are studied.

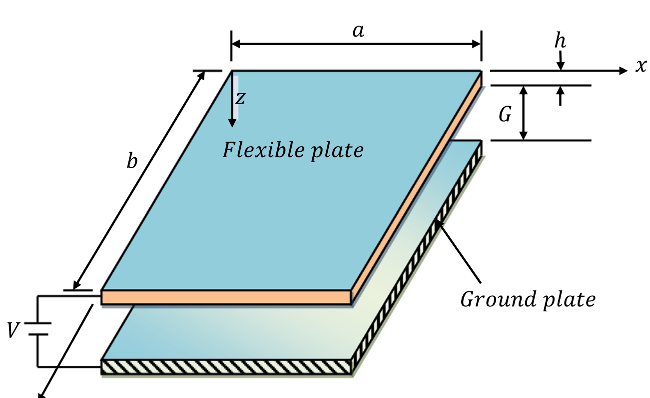

Fig. 1. Schematic of a nanoplate-based NEMS: kinematic parameters, coordinate system and geometry.

2. Governing equations and corresponding boundary conditions

A uniform nanoplate with the length a, width b and thickness h as depicted in Fig. 1 is considered. The initial air gap between the nanoplate and infinite ground plane is G. The nanoplate is subjected to the combined uniform hydrostatic force q_0 and non-uniform electrostatic force due to the applied voltage V. We can introduce a coordinate system (x, y, z) on one side of the mid-plane of the nanoplate. The upper and lower surfaces of the nanoplate at $z = +h/2$ and $z = -h/2$ are symbolized by $S^+$and S^-, respectively. The displacement components$(u_x,\ u_y,\ u_z)$ along the axes (x, y, z) can be written as

$$u_x = u(x,y) - z\frac{\partial w(x,y)}{\partial x}, \quad u_y = v(x,y) - z\frac{\partial w(x,y)}{\partial y}, \quad u_z = w(x,y). \tag{1}$$

where $u(x,y)$ and $v(x,y)$ are mid-plane displacements, $w(x,y)$ is the lateral deflection of the nanoplate. On the basis of the von-Karman hypothesis, the non-linear strain–displacement relations can be expressed as

$$\begin{aligned}&\varepsilon_{xx} = u_{,x} - zw_{,xx} + \tfrac{1}{2}w_{,x}^2, \quad \varepsilon_{yy} = v_{,y} - zw_{,yy} + \tfrac{1}{2}w_{,y}^2,\\ &\varepsilon_{xy} = \tfrac{1}{2}(u_{,y} + v_{,x} + w_{,x}w_{,y}) - zw_{,xy}.\end{aligned} \tag{2}$$

Based upon the linear elasticity, the stress components can be achieved as

$$\begin{aligned}&\sigma_{xx} = (\lambda + 2\mu)\left(u_{,x} + \tfrac{1}{2}w_{,x}^2 - zw_{,xx}\right) + \lambda\left(v_{,y} + \tfrac{1}{2}w_{,y}^2 - zw_{,yy}\right),\\ &\sigma_{yy} = (\lambda + 2\mu)\left(v_{,y} + \tfrac{1}{2}w_{,y}^2 - zw_{,yy}\right) + \lambda\left(u_{,x} + \tfrac{1}{2}w_{,x}^2 - zw_{,xx}\right),\\ &\sigma_{xy} = \mu(u_{,y} + v_{,x} + w_{,x}w_{,y} - 2zw_{,xy}).\end{aligned} \tag{3}$$

where parameters λ and μ show the classical Lame constants and can be obtained through

$$\lambda = \frac{E\nu}{1-\nu^2}, \quad \mu = \frac{E}{2(1+\nu)} \tag{4}$$

here E and ν denote the Young's modulus and Poisson's ratio, respectively. Since the classical continuum mechanics are unable to consider the atomic features of nanostructures, they must be modified; one efficient method is based on the Gurtin–Murdoch theory that incorporates size-effects into the conventional continuum approach. Owing to the fact that there is always interaction between the elastic surface and bulk material, the nanoplate is mostly undergoing in-plane loads in various directions which leads to surface stresses. According to the Gurtin–Murdoch theory these surface stresses can be computed by employing following surface constitutive equations

$$\begin{aligned}&\sigma^s_{\alpha\beta} = \tau^s\delta_{\alpha\beta} + (\tau^s + \lambda^s)\varepsilon_{\gamma\gamma}\delta_{\alpha\beta} + 2(\mu^s - \tau^s)\varepsilon_{\alpha\beta} + \tau^s u^s_{\alpha,\beta} \ (\alpha,\ \beta = x, y)\\ &\sigma^s_{\alpha z} = \tau^s u^s_{z,\alpha}\end{aligned} \tag{5}$$

where τ^s is the surface residual stress parameters. Also, λ^s and μ^s are the surface Lame constants and can be obtained through

$$\lambda^s = \frac{E^s\upsilon^s}{1-\upsilon^{s2}}, \quad \mu^s = \frac{E^s}{2(1+\upsilon^s)} \tag{6}$$

here and E^s and υ^s are the surface elastic modulus and surface Poisson's ratio, respectively.

Accordingly, the surface stress constituents at the upper and lower surfaces of the nanoplate can be achieved as

$$\sigma^{s\pm}_{xx} = (\lambda^s + 2\mu^s)\left(u_{,x} + \frac{1}{2}w_{,x}^2 \mp \frac{h}{2}w_{,xx}\right) + (\lambda^s + \tau^s)\left(v_{,y} + \frac{1}{2}w_{,y}^2 \mp \frac{h}{2}w_{,yy}\right)$$

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