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# Periodic response of bimodular laminated composite cylindrical panels with and without geometric nonlinearity

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ABSTRACT

The combined influence of bimodularity, geometric nonlinearity and curvature on the dynamic response characteristics of bimodular material laminated composite cylindrical panels is investigated under periodic excitation. The analysis is carried out using Bert's constitutive model and first order shear deformation theory based finite element. The geometrically nonlinear forced periodic response is obtained using shooting technique coupled with Newmark time marching, arc length/pseudo-arc length continuation algorithms. The effect of bimodularity, curvature, aspect ratio ($a/b$), radius to thickness ratio ($r/h$), boundary conditions, lamination scheme and small initial imperfection on the forced vibration response is analysed. The frequency response curves reveal significant difference in the positive and negative half cycle amplitudes for bimodular material panels which increases with the increase in curvature and bimodularity ratio. The through the thickness variations of normal strain/stress are presented to show the extent of assignment of tensile/compressive properties and restoring force due to combined influence of bimodularity and geometric nonlinearity. Period doubling and sub/super harmonic participations are observed for some cases.



## 1. Introduction

Many of the composites such as aramid–rubber, polyster–rubber, carbon–carbon, soft biological tissues, bone etc exhibit different elastic properties in tension and compression, and are referred to as bimodular composite materials [1–3]. A flat laminate of unimodular material symmetrically laminated about its middle-plane does not exhibit any bending–stretching coupling within the hypothesis of small deflection linear theory. On the other hand, a symmetrically laminated plate of bimodular material depicts bending–stretching coupling even when undergoing small deflections. The presence of initial middle surface curvature of panels, undergoing small transverse deflections, results in restoring membrane forces of tensile nature in the outward motion and compressive one in inward motion. For inward motion with finite deflection, the local curvature may dominate the initial curvature leading to compressive membrane forces acting in the destabilizing sense. Thus with the increasing inward deflection of thin walled curved panels, the restoring action of the membrane forces decreases, vanishes, may reverse and finally may again be restored due to tensile nature of membrane forces at large inward amplitudes. The combined influence of bimodularity, geometric nonlinearity and middle surface curvature is expected to significantly alter the dynamic behavior of bimodular panels as compared to unimodular panels. The necessity of analysing non-linear dynamics of bimodular material laminated composite panels arises not only from the requirements of effective/safe dynamic design, noise and vibration control of structural components used in automobiles, aerospace, missiles, rockets, biomechanical systems etc. but also as challenging problem exhibiting interesting nonlinear dynamic response characteristics.

In spite of the importance of dynamic analysis of bimodular laminated panels, the literature on free/forced vibration characteristics of bimodular structures is limited. The axisymmetric free vibration analysis of single-layer transversely isotropic circular/annular plates has been carried out using first-order shear deformation theory by Doong and Chen [4] and Chen and Juang [5]. Bert et al. [6], Doong and Chen [7] have carried out the free vibration analysis of single layer orthotropic and two layered cross-ply bimodular laminated composite rectangular plates. It is concluded that the fundamental natural frequency increases with the increase in bimodularity ratio. The free vibration analysis of bimodular laminated angle ply plates has been carried out by Patel et al. [8] based on higher order shear deformation. Bert and Kumar [9] and Khan et al. [10] have carried out the free vibration analysis of cross- and angle-ply bimodular laminated cylindrical panels, respectively. Positive and negative half cycle frequencies are found to be significantly different depending upon the bimodularity ratio, geometrical parameters and boundary conditions. The transient analysis of bimodular rectangular plates has been

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carried out by Reddy [11] and Patel et al. [12] for first few response cycles. Steady state frequency response analysis of bimodular laminated cylindrical panels has been carried out by Khan et al. [13]. Zhang et al. [14] have employed parametric variational approach [15] for the transient dynamic analysis of isotropic bimodular structures. The effect of geometric non-linearity is not considered in the dynamic analyses of bimodular structures dealt in the literature except Ref. [16] dealing with the nonlinear dynamics of bimodular laminated plates wherein hardening non-linearity was observed.

To the best of the authors' knowledge, the dynamic behavior of laminated bimodular composite cylindrical panels considering geometric nonlinearity has not been investigated. The consideration of geometric nonlinearity becomes important especially for thin panels undergoing vibrations with amplitude of the order of their thickness wherein linear analysis may not be adequate for predicting the deflections, strains, stresses and frequencies to the desired level of accuracy. The presence of initial middle surface curvature of panels coupled with bimodularity and geometric nonlinearity is expected to depict softening/hardening nonlinearity depending on the parameters.

The combined influence of bimodularity, geometric non-linearity and curvature on the frequency response of bimodular laminated cylindrical panels subjected to harmonic excitation is investigated for the first time in this paper. It is apt to make a mention here that the steady state response analysis of bimodular laminates is a challenging task owing to sudden changes in restoring force from positive/negative half cycle to negative/positive half cycle which may induce numerical instability. The study is carried out using field consistent eight-noded isoparametric finite element based on first-order shear deformation theory and Bert's constitutive model. Newmark's time integration coupled with Newton Raphson method is used to solve the governing equations in time-domain. The steady state frequency response characteristics are obtained using shooting technique and arc-length/pseudo arc-length continuation methods. The temporal and through the thickness stress/strain variations are presented to explore the non-linear dynamic behavior of bimodular laminated composite cylindrical panels. With the inclusion of geometric non-linearity, the frequency response reveals softening and hardening type of nonlinear behaviour depending upon the curvature and boundary conditions of panels.

## 2. Formulation

The geometry and coordinate system of a laminated composite cylindrical panel (radius $r$, meridional length $a$, circumferential length $b$ and total thickness $h$) are shown in Fig. 1. The displacement field $(u, v, w)$ at a point $(s, \theta, z)$ is expressed as function of middle surface displacements $u_0$, $v_0$, $w_0$ and independent rotations $\beta_s$ and $\beta_\theta$ of the meridional and hoop sections, respectively, as:

$$
\begin{aligned}
u(s,\theta,z,t) &= u_0(s,\theta,t) + z\,\beta_s(s,\theta,t) \\
v(s,\theta,z,t) &= v_0(s,\theta,t) + z\,\beta_\theta(s,\theta,t) \\
w(s,\theta,z,t) &= w_0(s,\theta,t)
\end{aligned} \tag{1}
$$

The spatial variation of displacement field variables is expressed in terms of their nodal values using a $C^\circ$ continuous eight-noded serendipity quadrilateral element as:

$$
(u_0, v_0, w_0, \beta_s, \beta_\theta) = \sum_{i=1}^{8} N_i^0 (u_{0i}, v_{0i}, w_{0i}, \beta_{si}, \beta_{\theta i}) \tag{2}
$$

where $N_i^0$ are the original shape functions of the element. To avoid membrane/transverse shear locking, the constrained strain terms are interpolated using modified shape functions [17].

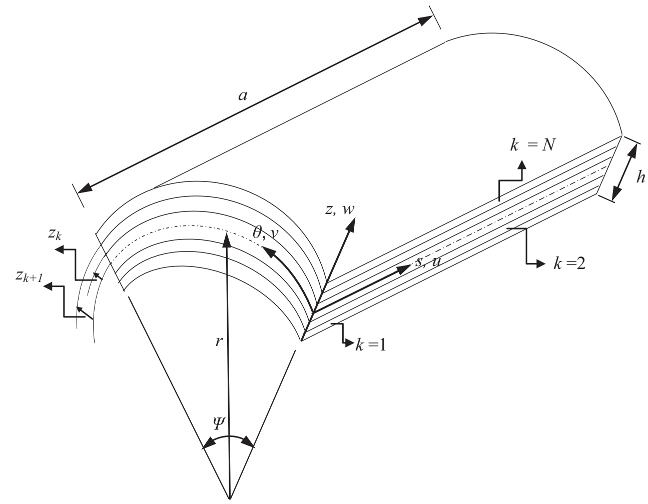


**Fig. 1.** Geometry and coordinate system of cylindrical panel.

Using Sanders' shell theory, strain field in terms of mid-surface deformation variables can be written as

$$
\{\varepsilon\} = \begin{Bmatrix} \varepsilon_{ss} \\ \varepsilon_{\theta\theta} \\ \gamma_{s\theta} \\ \gamma_{sz} \\ \gamma_{\theta z} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{\varepsilon}_\mathbf{P}^\mathbf{L} \\ \mathbf{0} \end{Bmatrix} + \begin{Bmatrix} z\boldsymbol{\varepsilon}_\mathbf{b} \\ \boldsymbol{\varepsilon}_\mathbf{s} \end{Bmatrix} + \begin{Bmatrix} \boldsymbol{\varepsilon}_\mathbf{P}^\mathbf{NL} \\ \mathbf{0} \end{Bmatrix} \tag{3}
$$

where linear mid-surface membrane $\boldsymbol{\varepsilon}_\mathbf{P}^\mathbf{L}$, bending $\boldsymbol{\varepsilon}_\mathbf{b}$, transverse shear $\boldsymbol{\varepsilon}_\mathbf{s}$ and the non-linear mid-surface membrane $\boldsymbol{\varepsilon}_\mathbf{P}^\mathbf{NL}$ strain vectors are defined as [18]:

$$
\boldsymbol{\varepsilon}_\mathbf{P}^\mathbf{L} = \begin{Bmatrix} \frac{\partial u_0}{\partial s} \\ \frac{\partial v_0}{r\partial\theta} + \frac{w_0}{r} \\ \frac{\partial u_0}{r\partial\theta} + \frac{\partial v_0}{\partial s} \end{Bmatrix}, \quad \boldsymbol{\varepsilon}_\mathbf{b} = \begin{Bmatrix} \frac{\partial\beta_s}{\partial s} \\ \frac{\partial\beta_\theta}{r\partial\theta} \\ \frac{\partial\beta_s}{r\partial\theta} + \frac{\partial\beta_\theta}{\partial s} \end{Bmatrix}, \quad \boldsymbol{\varepsilon}_\mathbf{s} = \begin{Bmatrix} \beta_s + \frac{\partial w_0}{\partial s} \\ \beta_\theta + \frac{\partial w_0}{r\partial\theta} - \frac{v_0}{r} \end{Bmatrix}, \quad \boldsymbol{\varepsilon}_\mathbf{P}^\mathbf{NL} = \begin{Bmatrix} \frac{1}{2}\left(\frac{\partial w_0}{\partial s}\right)^2 \\ \frac{1}{2}\left(\frac{\partial w_0}{r\partial\theta} - \frac{v_0}{r}\right)^2 \\ \frac{\partial w_0}{\partial s}\left(\frac{\partial w_0}{r\partial\theta} - \frac{v_0}{r}\right) \end{Bmatrix} \tag{4}
$$

The fiber-governed constitutive relations for an arbitrary layer '$k$' in the laminate $(x, y, z)$ coordinate system can be expressed as

$$
\left\{\boldsymbol{\sigma}^k\right\} = \left\{ \sigma_{ss}^k \quad \sigma_{\theta\theta}^k \quad \tau_{s\theta}^k \quad \tau_{sz}^k \quad \tau_{\theta z}^k \right\}^\mathrm{T} = \left[\overline{\mathbf{Q}}_{kl}\right]\{\boldsymbol{\varepsilon}\} \tag{5}
$$

where $\boldsymbol{\sigma}^k$ is stress field in the $k$th layer with meridional ($\sigma_{ss}^k$), hoop ($\sigma_{\theta\theta}^k$), in-plane shear ($\tau_{s\theta}^k$) and transverse shear ($\tau_{sz}^k$, $\tau_{\theta z}^k$) components; $\overline{\mathbf{Q}}_{kl}$ is the transformed stiffness coefficients matrix, the subscript $l$ refers to the bimodular characteristics: $l=1$ and 2 denote the properties associated with fiber direction tension and compression, respectively [3].

The kinetic energy of the panel is given by

$$
T = \frac{1}{2}\iint \left[ \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \rho_k \{ \dot{u}_k \quad \dot{v}_k \quad \dot{w}_k \} \{ \dot{u}_k \quad \dot{v}_k \quad \dot{w}_k \}^\mathrm{T} \mathrm{d}z \right] r\mathrm{d}s\mathrm{d}\theta \tag{6}
$$

where $\rho_k$ is the mass density of the $k$th layer and $z_k$, $z_{k+1}$ are the $z$-coordinates of the laminate corresponding to the inner and outer surfaces of the $k$th layer, and $N$ is the number of layers.

The total potential energy functional $U$ consisting of strain energy and potential of uniformly distributed transverse load is given by

$$
U = \frac{1}{2}\iint \left[ \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \left\{\boldsymbol{\sigma}^k\right\}^\mathrm{T} \{\boldsymbol{\varepsilon}\} \mathrm{d}z \right] r\mathrm{d}s\mathrm{d}\theta - \iint q w_0 r\mathrm{d}s\mathrm{d}\theta \tag{7}
$$

where $q$ is distributed transverse load.