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# Efficient meshless SPH method for the numerical modeling of thick shell structures undergoing large deformations

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ABSTRACT

The objective of this paper is to present an extension of the Lagrangian Smoothed Particle Hydrodynamics (SPH) method to solve three-dimensional shell-like structures undergoing large deformations. The present method is an enhancement of the classical stabilized SPH commonly used for 3D continua, by introducing a Reissner–Mindlin shell formulation, allowing the modeling of moderately thin structure using only one layer of particles in the shell mid-surface. The proposed Shell-based SPH method is efficient and very fast compared to the classical continuum SPH method. The Total Lagrangian Formulation valid for large deformations is adopted using a strong formulation of the differential equilibrium equations based on the principle of collocation. The resulting non-linear dynamic problem is solved incrementally using the explicit time integration scheme, suited to highly dynamic applications. To validate the reliability and accuracy of the proposed Shell-based SPH method in solving shell-like structure problems, several numerical applications including geometrically non-linear behavior are performed and the results are compared with analytical solutions when available and also with numerical reference solutions available in the literature or obtained using the Finite Element method by means of ABAQUS© commercial software.



## 1. Introduction

After more than half a century of development, Finite Element Method (FEM) has become the most popular and widely used numerical method in the analysis of real world engineering problems associated with large systems of Partial Differential Equations (PDEs). Typically in such an analysis the continuum or region of interest is partitioned into a finite number of elements which have nodal points at their vertices. The solution is then approximated locally on every element independently using shape functions. This local character makes PDEs easy to solve using the FEM and therefore enhance the robustness of this method; nevertheless, it still suffers from some drawbacks. For instance, in the problems involving high gradients variations such as metal forming and crashworthiness, the FEM may diverge unless a very fine mesh is used and consequently a very small time step is required. Another challenging domain concerns the crack propagation phenomenon which needs remeshing and mapping operations which are numerically difficult and tedious. All of these shortcomings make the FEM computationally expensive and not really useful especially in complex industrial problems.

To avoid the aforementioned problems, researchers have been developing other alternative methods called "*meshless*" or "*particle*" methods, such as the Smoothed Particle Hydrodynamics (SPH) method [1,2], the Element Free Galerkin (EFG) method [3], the Reproducing Kernel Particle Method (RKPM) [4], and the Finite Point Method (FPM) [5]. Unlike the conventional grid-based FEM, in these meshless methods, only particles or nodes are generated and scattered to represent the continuum shape and special continuous weighting functions are defined in a compact support domain for each point. They are adaptive for the changes of the topological structure of a continuum and efficient to handle crack growth, explosion, *etc.* [3,6–8].

The SPH represents one of the most efficient meshless methods; it has been invented initially for the study of astrophysical phenomena [1,2]. The SPH method is based on the principle of collocation using strong formulation directly applied to the PDEs of the problem. Because of its strong ability to incorporate complex physical phenomena, the SPH method is nowadays being extended to the problems of computational fluid and solid mechanics [9,10]. Monagan and Kocharyan [11] extended the general SPH formulation to deal with two phase-flow of a dusty gas. Cleary [12] developed the SPH method to describe accurately the conductive and convective

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heat transfer for a sequence of idealized benchmark problems. Alia and Souli [13] used the Eulerian multi-material formulation to simulate high pressure wave propagation in the air explosion. Zhang and Batra [14] proposed a Modified Smoothed Particle Hydrodynamics (MSPH) method to study one-dimensional wave propagation and two-dimensional transient heat conduction problems. Plane strain forging process was simulated using constant Corrected Smoothed Particles Hydrodynamics (CSPH) [15], in which the metal was regarded as non-Newtonian fluid. Complex 3D flow in high pressure die casting, extrusion and forging metal forming processes were carried using 3D SPH program [16]. SPH method with different expressions of artificial viscosity was studied in the high-velocity impacts of metallic projectiles on thin metallic plates [17]. The effects of different detonation cavity angles and different charge head lengths are investigated using SPH method by Liu et al. [18].

The classical SPH method has been developed and successfully applied in structural mechanics as well as in the modeling of forming processes, in metal cutting and in impact on a fuel tank using only three-dimensional continuum approach [17,19]. Yet modeling shell-like structures using the classical 3-D continuum SPH approach is very time-consuming, because several particles need to be employed in the thickness direction. It has been shown [19] that a minimum of three particles are needed through the plate thickness to ensure a good quality of results. Hence, simple and time-saving meshless shell formulation with a single layer of particles on the mid-plane has attracted numbers of researchers. For instance, Krysl and Belytschko [20] combined the Kirchhoff plate theory with EFG method for solving structural problems involving plates, in which a congruent background cells are necessary to integrate the global weak forms. Li et al. [21] adopted the RKPM to simulate large deformations of thin shell structures using the window function to construct highly smoothed shape functions. The Meshless Local Petrov–Galerkin method was successfully used for solving Mindlin shells by using a local weak form [22]. SPH method was also developed by Maurel et al. [23], Caleyron et al. [24] for thick shell modeling based on Moving Least Square (MLS) approximation, which is also applied in solids and irregular structures [25,26]. However, since the MLS method does not exhibit the Kronecker delta property, hence it cannot allow imposing directly the essential boundary conditions (Dirichlet conditions). To accurately describe the essential boundary conditions, several rows of particles (ghost particles) have to be applied to extend the fixed boundary, which makes the procedure complex and with more computation efforts.

The aim of the present paper is to study and evaluate a shell-based SPH method based on the Reissner–Mindlin theory, for the non-linear quasi-static solutions of shell structures undergoing very large deformations but with small strains. Though, some drawbacks such as the lack of consistency, tensile instability and zero energy modes in the numerical computation limit the application of the classical SPH method using an Updated Lagrangian (UL) formulation [27–29]. To restore the linear consistency, we adopt the Corrective Smoothed Particle Method (CSPM) [30] which is suitable for modeling any unsteady boundary value problems with the Dirichlet and/or von Neumann types of boundary conditions. It also addresses the tensile instability problem by enforcing the higher order consistency. A Total Lagrangian (TL) formulation with respect to the initial configuration is also employed to reduce the tensile instability problem [31]. Moreover an artificial viscosity expression was introduced [32] in order to temper the oscillations of the numerical solution especially in the shocking regions.

The outline of this paper is as follows. In the next section, the kinematics of the shell structure discretized using SPH particles in the mid-surface is developed together with the constitutive relations for the case of elastic homogenous materials. In Section 3 are developed the coupling between the CSPM and the TL formulation which is valid for large deformations of solids. This combined formulation allows a robust and fast shell-based SPH modeling of thin structures. Section 4 extends the TL formulation of the SPH for the modeling of shell structures by employing only one layer of particles in the mid-surface of the shell, the resulting generalized equations of shells are derived using a strong formulation of the differential equilibrium equations. Several numerical applications involving large deformations of moderately thin shell structures are presented in Section 5. Finally, some concluding remarks are drawn in Section 6.

## 2. Governing equations of shell structures using SPH method

### 2.1. Kinematics

To establish an adaptive SPH formulation for thin or thick shell structures, the Reissner–Mindlin theory has to be considered, in order to take into account the transverse shear stresses which may become important for relatively thick shells. According to this theory, the shell structure behavior can be represented by using only one layer of particles at the mid-surface of the shell (see Fig. 1). Each SPH particle possesses five degrees of freedom: three translations $\mathbf{u}_L = \{u, v, w\}^T$ and two rotations $\boldsymbol{\theta}_L = \{\theta, \varphi\}^T$ expressed in the local framework tangent to the shell mid-surface. Passing through a particle, the straight transverse fiber normal to the mid-surface remains straight but not necessarily perpendicular to the mid-surface after deformation. This fiber is called a pseudo-normal vector generally represented by $\boldsymbol{n}$ (Fig. 1), especially denoted by $\boldsymbol{n}_0$ in the initial configuration.

Let suppose the shell structure given in Fig. 1, the shell is assumed to have a uniform thickness $t$, the position vector $\boldsymbol{x} = \boldsymbol{x}_q$ of any material point $q$ located at a distance $\zeta$ from the shell mid-surface, can be expressed as

$$\boldsymbol{x}(\xi,\ \eta,\ \zeta) = \boldsymbol{x}_p(\xi,\ \eta) + \zeta\,\boldsymbol{n}(\xi,\ \eta), \quad \zeta \in \{-t/2,\ t/2\} \tag{1}$$

where $\boldsymbol{x}_p$ is the position vector of the point $p$ located on the mid-surface, $\boldsymbol{\xi} = (\xi,\ \eta,\ \zeta)$ is the position vector described in the curvilinear coordinates.

The displacement vector $\boldsymbol{u} = \boldsymbol{u}_q$ of point $q$ can be calculated by

$$\boldsymbol{u}(\xi,\ \eta,\ \zeta) = \boldsymbol{u}_p(\xi,\ \eta) + \zeta\,\Delta\boldsymbol{n}(\xi,\ \eta) \tag{2}$$

where $\Delta\boldsymbol{n} = \boldsymbol{n} - \boldsymbol{n}_0$.

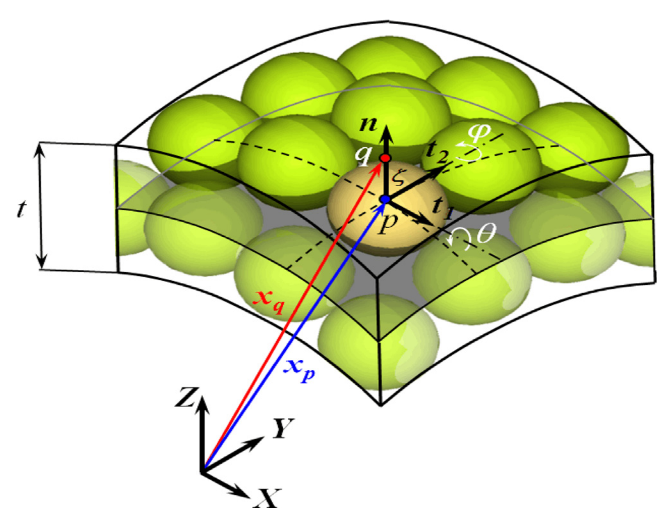


**Fig. 1.** Discretization of a shell mid-surface using SPH particles.