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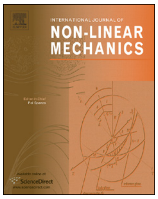

Theoretical approach on the dynamic global buckling response of metallic corrugated core sandwich columns

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ABSTRACT

A theoretical (*semi-analytical*) approach was proposed to estimate the dynamic in-plane response of corrugated core sandwich columns against suddenly applied loads with a compression rate less than 5 m/s. The model has been constructed so as to effectively include various dynamic effects such as stress wave propagation, material rate dependence and lateral inertia. The practical and theoretical complexities caused from the dynamic phenomena and the established governing equations (*e.g.* coupled non-uniform axial force distribution) have been resolved by employing Galerkin's method. The proposed approach was validated by comparing the calculations from the theoretical model and Finite Element Method (FEM): the load history and deformation shape of extruded Al6061-T6 corrugated core sandwich columns and bending/brazed SS304 corrugated core sandwich columns. The model successfully yielded the imperfection-sensitive, velocity-dependent dynamic response and appearance of higher buckling modes. In addition, it has been demonstrated that the sandwich columns with periodic cellular metals outperform their weight-equivalents, monolithic solid columns, under dynamic conditions. The proposed approach as an efficient tool to explore the dynamic global buckling response in design space can make preliminary studies of weight minimization for dynamic applications.

1. Introduction

A lot of attention has been given to metallic core sandwich structures with periodic cellular metals due to weight-saving and multifunctional benefits [1–4]. From a structural design viewpoint, the sandwich structures have a multitude of failure modes and design variables. For example, a corrugated core sandwich column under in-plane loading largely fails either by global buckling or by local buckling depending on geometric dimensions and material selection [5–7]. As a result, a lot of theoretical, numerical, and experimental studies have been conducted to find its maximum collapse load with their minimized weight [5–11].

A possible loading scenario of real-life-structures, suddenly applied dynamic loading, influences their structural response due to various dynamic effects such as inertia, material strain-rate dependence, appearance of higher buckling modes, and stress wave propagation, etc. For the last decade, the dynamic out-of-plane response of metallic core sandwich structures subject to impulsive loading has been actively investigated to maximize the energy absorption performance [12–22]. Accordingly, efforts have been made to investigate their deformation mechanisms and momentum transfer under impulsive loading and to identify the effects of metallic cores, specifically, the selection of core topology and relative core density, on the dynamic performance. Consequently, it is revealed that metallic core sandwich structures are superior to their weight-equivalents, monolithic structures [20–22].

On the other hand, the dynamic in-plane response of sandwich columns with periodic cellular metals has not received much attention [23]. Accordingly, the following must be understood in order to introduce those column structures into dynamic applications: the relationship between various sandwich design variables, failure mechanisms and their dynamic response. Even though FEM is a powerful tool to provide reliable numerical results, the amount of time and cost for modeling and calculation cannot be neglected due to their discrete core structures. Therefore, a simplified approach on a theoretical basis is required to explore its dynamic response associated with its geometric dimension and material properties.

In this study, we are going to focus on corrugated core sandwich columns made of Al6061-T6 and SS304 subject to dynamic in-plane loading. The main objective is to develop analytical models predicting dynamic *global buckling failure* in

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Nomenclature

Superscript

(m)	monolithic solid column
(SW)	corrugated core sandwich column
$(perp)$	perpendicular-to-corrugations
$(para)$	parallel-to-corrugations

Subscript

(el)	elastic
(pl)	plastic
ν	Poisson's ratio
ν_P	plastic Poisson's ratio (here, ν_P=0.5)
κ	Becque's parameter (the ratio of the plastic strain increment in the principal 2 direction to the plastic strain increment in the principal 1 direction)
ω	corrugation angle
$\overline{\rho}$	core relative density
ρ	parent material density
σ	material stress
σ_Y	material stress
ε	material strain
ξ	the imperfection amplitude of global curvature
$\varphi^{(SW)}$	rotation due to bending moments
A	cross sectional material area
c	core height
$c_{(\cdot)}^{(\cdot)}$	elastic or elastic–plastic wave speed of columns such as monolithic solid columns, corrugated core sandwich columns compressed perpendicular-to-corrugations or parallel-to-corrugations
D	total flexural rigidity of a sandwich column
$D_{(\cdot)}^{(\cdot)}$	flexural rigidity of elastic or elastic–plastic columns such as monolithic solid columns, corrugated core sandwich columns compressed perpendicular-to-corrugations or parallel-to-corrugations
d	separation between face sheet centroids
D_f	flexural rigidity of the sandwich column face about its own axis

E	Young's modulus
E_P	plastic modulus
$\mathrm{E_T}$	tangent modulus
$E_T^{'}$	the tangent modulus of the plane strain true stress versus logarithmic strain curve
G_c	core shear modulus
$G_{c,(\cdot)}^{(\cdot)}$	core shear modulus of elastic or elastic–plastic columns such as monolithic solid columns, corrugated core sandwich columns compressed perpendicular-to-corrugations or parallel-to-corrugations
h	face sheet thickness
L	column span length
Δl_{ext}	the amount of column extension due to transverse deflection
Δl_{comp}	the amount of column shortening due to an end compression
l	core ligament length
P	axial force
P_1	axial force distribution due to an end compression
P_2	axial force distribution due to a change of transverse column deflection
$P_{cr}^{(QS)}$	quasi-static critical failure load
$\dot{p}$	equivalent plastic strain rate
R	the dynamic yield strength enhancement ratio
S	core shear rigidity
$S_{(\bullet)}^{(\bullet)}$	shear rigidity of elastic or elastic–plastic columns such as monolithic solid columns, corrugated core sandwich columns compressed perpendicular-to-corrugations or parallel-to-corrugations
t	core ligament thickness, or time
V	compression velocity, rate-of-compression
w_0	global curvature imperfection
$w^{(m)}$	transverse displacement of monolithic solid column from the undeformed initial curved shape
$w^{(SW)}$	transverse displacement of corrugated core sandwich column from the undeformed initial curved shape
$w_s^{(SW)}$	displacement due to transverse shear force
$W(x)$	base functions (Eigen functions) for column deformation

corrugated core sandwich columns and their weight-equivalents, monolithic solid columns. The analytical model for the prediction of dynamic global buckling of corrugated core sandwich columns is developed from knowledge derived from studies on monolithic solid columns [23–30]. Finally, the models will be validated by comparison with the Finite Element Method (FEM) through some examples.

2. Problem statements

2.1. Problem definition

Three column structures are studied: compression of a monolithic solid column and compression of corrugated core sandwich columns in two in-plane loading directions, as illustrated in Fig. 1. For all these problems, one end (front end) is axially displaced at a constant speed of V with the other degrees of freedom (DOFs) constrained while the other end (back end) is completely fixed. The column material is either Al6061-T6 or SS304, which can be modeled as a *rate-independent* elastic–plastic material and a *rate-dependent* one, respectively. All the columns are assumed to have initial curvature imperfections in the form of the fundamental static buckling mode, $w_0(x) = (\xi/2L)(1 - \cos(2\pi x/L))$.

In this study, the considered range of applied velocity, V, is $V/c_{el}\epsilon_Y < 1$ such that the incident stress wave at the moment of imposing compression is purely elastic.

The sandwich parameters of a corrugated core sandwich column are indicated in Fig. 1(b) and (c). Due to employed manufacturing methods, the geometries of Al6061-T6 and SS304 corrugated cores differ slightly [5,6]; the former is made using an extrusion/friction stir weld technique, and the latter is manufactured using a bending/brazing method. Therefore, the core relative density, $\overline{\rho}$, which is the ratio of core density to the density of parent material of which the column is made, is dependent on the manufacturing methods [5,6] and can be calculated using Eqs. (1) and (2).

$$\overline{\rho}_{Al} = \frac{t/\sin\omega}{\ell\cos\omega + t/\sin\omega} \tag{1}$$

$$\overline{\rho}_{SS304} = \frac{2t}{\ell\sin 2\omega} \tag{2}$$

The parent material properties of Al6061-T6 and SS304 used in this study are based on reported test data [5,6,31–33]. The