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Dynamic analysis of a dielectric elastomer-based microbeam resonator with large vibration amplitude

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ABSTRACT

A non-linear vibration equation with the consideration of large amplitude, gas damping and excitation is developed to investigate the dynamic performance of a dielectric elastomer (DE)-based microbeam resonator. Approximate analytical solution for the vibration equation is obtained by applying parameterized perturbation method (PPM) and introducing a detuning variable. The analysis exhibits that active tuning of the resonant frequency of the resonator can be achieved through changing an applied electrical voltage. It is observed that increasing amplitude will increase the natural frequency while it will decrease the quality factor of the resonator. In addition, it is found that the initial pre-stretching stress and the ambient pressure can significantly alter the resonant frequency of the resonator. The analysis is envisaged to provide qualitative predictions and guidelines for design and application of DE-based micro resonators with large vibration amplitude.

1. Introduction

Driven by its broad spectrum of potential applications, dielectric elastomer (DE), an emerging electroactive material capable of producing large deformation [1,2], has been extensive explored during the past decades. Except the capability of large deformation, DE also possesses an excellent combination of flexibility, low cost, and chemical and biological compatibility, which makes DE a promising material candidate for artificial muscles, energy harvesters, soft robots, programmable haptic surfaces and resonators [2–5]. As material for micro resonators, DE has been considered as a more efficient and superior alternative compared to conventional materials, such as silicon, due to the capability of active tuning of resonant frequency after fabrication without adding extra exterior components [6–8]. The typical structure of a DE-based resonator usually consists of a thin layer of pre-stretched DE membrane sandwiched between two facing electrodes. When a static electrical voltage is applied between the two electrodes, an electrostatic compression is generated altering the resonant frequency by relaxing the tension of the sandwiched DE structure.

The unique attributes of DE as mentioned have been drawing great interests in using DEs as materials for resonators recently. For example, Dubois et al. [9] experimentally demonstrated the feasibility of frequency tuning of a circular membrane resonator made of polydimethylsiloxane (PDMS) elastomer. By studying the time response of a spherical membrane resonator, Mockensturm and Goulbourne [10] found that the performance of the membrane could be controlled by applying an electrostatic pressure. Zhu et al. [11] analyzed the oscillation of a spherical DE membrane resonator under harmonic, subharmonic and superharmonic resonance and exhibited the ability of frequency tuning. Zhang et al. [8] successfully fabricated a microbeam resonator with polymer structure and their experimental investigation showed that the ambient pressure significantly decreased the quality factor (Q-factor) of the resonator. By establishing a governing equation with considering squeeze-film damping, Feng et al. [12] investigated the dynamic performance of a DE-based micro-beam resonator under small amplitude vibration. Recently, Li et al. [13] studied the dynamic response and the stability of a DE resonator and presented that the pre-stretch and the applied voltage could tune the dynamic behavior of the resonators. Besides, there are some other works focusing on stability analysis [14–16]. Although efforts have been devoted to studying DE-based resonators, most of the previous studies were focused on oscillation with small amplitude. Nevertheless, one of the favorable attributes of DEs over other material candidates is large deformation, which can be activated by

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applying periodic electrical voltage. It has been reported that periodic voltage can induce strain in DE structures from a linear strain of 4% to over 100% [1,16,17]. For applications of DE-based resonators aiming at large deformation, it is natural to investigate the large amplitude effects upon the dynamic performance of the devices whereas limited work has been found on such the effects. Owning the benefit of large deformation, DE-based devices are susceptible to instability [14,18]. Fortunately, it has been suggested that the instability could be avoided by spaying charges on the DE membrane [15,19]. Therefore, in this current work the large deformation effects will be studied without involving any possibility of instability.

As demonstrated by Zhang et al. [8] gas damping can influence the performance of the resonator, i.e., the Q-factor decreases significantly with the increase of the ambient pressure. Even though micro devices are usually sealed in gas-evacuated cavities to reduce gas damping, it was commented by Yasumura et al. [20] and Ho and Tai [21] that gas damping for micro devices cannot be neglected when the ambient pressure is above 10^{-6} atm. Therefore, the consideration of gas damping under large amplitude vibration is necessary and essential for accurate prediction on the dynamic performance of DE-base resonators in non-vacuum cavity.

In this current work, based on a typical configuration of the DE-based resonator fabricated by Zhang et al. [8] (shown in Fig. 1), a non-linear governing equation is established to study the dynamic performance of the resonator. Parameterized perturbation method (PPM) and a detuning variable are introduced to obtain an approximate analytical solution of the governing equation. Then the analytical results are compared to numerical results and existing linear results [12] to validate the approaches and observed the large amplitude effects.

2. Governing equations

Compared to small amplitude vibration, when the beam in Fig. 1 vibrates with large amplitude, the midplane of the beam will be elongated instead of remaining un-stretched. The elongation will alter the effective rigidity of the beam and influence the vibration behavior of the resonator correspondingly. In addition, the gas damping involved in this current work will not only depends on the dimensions of the structure and the properties of the gas as that for small amplitude vibration [12], but also will depend on the vibration amplitude. Therefore, the gas damping under large amplitude vibration needs to be evaluated.

2.1. Non-linear vibration equation

When the double-clamped beam in Fig. 1 is subjected to an external load i.e., $q(x,t)$, the beam with length, width and height being l, b and d, respectively, will deflect laterally. Fig. 2 shows the deformation of a differential element of the beam.

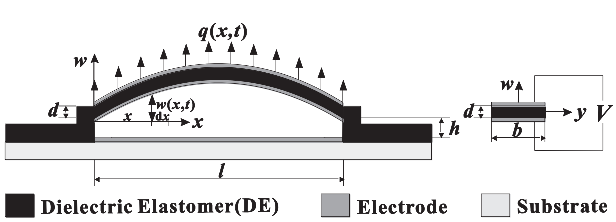

Fig. 1. Schematic demonstration of a double-clamped microbeam with deformation.

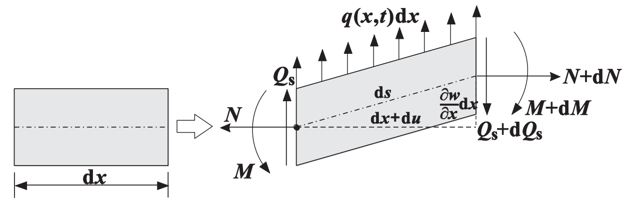

Fig. 2. Schematic demonstration of deformation of a differential element.

Under steady-state oscillation, the equilibrium equation of forces for the differential element in the vertical direction is

$$Q_S + q(x,t)\mathrm{d}x - (Q_S + \mathrm{d}Q_S) + \rho A\frac{\partial^2 w}{\partial t^2}\mathrm{d}x = 0 \tag{1}$$

where Q_S is the shear force; ρ is the density of the beam; $A=bd$ is the area of the beam cross section; w is the displacement in the vertical direction and t is time. Furthermore, the balance of bending moments yields

$$M + \mathrm{d}M + \frac{1}{2}q(x,t)(\mathrm{d}x)^2 + Q_S\mathrm{d}x + N\mathrm{d}w - M = 0 \tag{2}$$

where M is the bending moment and N is the axial force along the horizontal direction. Combining Eqs. (1) and (2) and neglecting the higher order of the infinitesimal term, we can have the following vibration equation:

$$EI\frac{\partial^4 w}{\partial x^4} - N\frac{\partial^2 w}{\partial x^2} + \rho A\frac{\partial^2 w}{\partial t^2} - q(x,t) = 0 \tag{3}$$

with boundary conditions

$$w(0,t) = w(l,t) = 0, \frac{\partial w(0,t)}{\partial x} = \frac{\partial w(l,t)}{\partial x} = 0 \tag{4}$$

where E is the Young's modulus of the beam and $I=bd^3/12$ is the second moment of the beam cross section. For forced vibration, $q(x,t)$ in vibration Eq. (3) can be written as $q(x,t)=q_d+q_e$ with q_d and q_e denoting the damping and the excitation forces loaded on unit length of the beam, respectively. Here it should be noted that except the axial forces considered for small amplitude [12], the axial force N in Eq. (3) for large amplitude needs to incorporate the force induced by elongation of the beam midplane. From Fig. 2 showing the deformation, the elongation of the midplane for the differential element is

$$e = \mathrm{d}s - \mathrm{d}x = \left(\sqrt{\left(1+\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2} - 1\right)\mathrm{d}x \tag{5}$$

where ds is the length of the element after deformation. By integrating the expansion of Eq. (5) over the beam length and neglecting the higher order of the infinitesimal term, the elongation of the midplane of the beam is derived as

$$\Delta = w(l) - w(0) + \frac{1}{2}\int_0^l \left(\frac{\partial w}{\partial x}\right)^2 \mathrm{d}x \tag{6}$$

where $w(0)$ and $w(l)$ are the transverse displacements of the two ends of the beam, particularly $w(0)=w(l)=0$ for a double-clamped beam. Accordingly, the axial force N in Eq. (3) for large deformation is

$$N = \left(\sigma_0 - \varepsilon_0\varepsilon_r\frac{V^2}{d^2} + \frac{E\Delta}{l}\right)A \tag{7}$$

where σ_0 is the initial pre-stretching stress; ε_0 and ε_r are the dielectric permittivity of vacuum and the relative dielectric permittivity of the elastomer, respectively, and V is the applied static voltage across the beam.

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