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Unwinding motion of cable by taking into consideration effect of bending on cable

Kun-Woo Kim [1], Wan-Suk Yoo *

School of Mechanical Engineering, Pusan National University, Busan 609-735, South Korea

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ABSTRACT

One of the most important factors influencing the performance of wire-guided missiles is the ability to exchange information through a connected cable. Some of the problems that occur during the process of unwinding the cable from the spool package are tangling of the cable because of low tensile force at the guide-eyelet point and cutting of the cable because of high tensile force. Therefore, it is important to analyze the transient motion of the unwinding cable withdrawn from the spool package, while considering the effect of flexural rigidity on the cable. The unwinding system is defined with cylindrical coordinates, and Hamilton's principle in an open system is introduced to represent the mass change of the cable in the control volume. Using Hamilton's principle, which takes into consideration the Lagrangian, virtual work, and virtual momentum transport, the unwinding equation with high non-linearity can be derived using boundary conditions. Further, by assuming inextensibility, the tensile force can be derived for the spatial variable. To solve the unwinding equation numerically, Newmark implicit integration is utilized with the central finite-difference approximation for spatial variables. The motions of the cable can be ascertained on the basis of various initial tensile forces by considering the constant unwinding velocity at the guide-eyelet point in air and water. It can be concluded from these motions that fluid resistance is the dominant force on the cable when it is unwound in the water, and, in contrast, centrifugal force is dominant in the air. Further, it can be observed that the initial tensile force required when the cable is unwound in water is greater than that required in air, to avoid unwinding problems such as tangling and cutting of the cable.

1. Introduction

Missiles and torpedoes must be in continuous communication with a central control system. Instructions on military actions are imparted to these weapons through wire-guided cable, which can have a definite impact on target performance. As a result, the analysis of the unwinding motion of wire-guided cable has become an important issue. Cables are tens kilometer long, therefore, problems such as tangling or cutting can occur frequently when they are unwound from the spool package. In such cases, the target performance of missiles or torpedoes will be adversely affected, leading to a significant loss in the fighting capability during wartime and the state of emergency. Therefore, it is important to analyze and investigate the motion of a cable when it is unwound from a spool package, and to use the results of this analysis when designing spool packages and selecting types of cable.

The analysis of the unwinding motion for a cable was introduced by Padfield [1] in 1956 in the field of textile engineering. She studied the unwinding motion of yarn using the shooting method from the steady-state equation of motion defined in an orthogonal coordinate system, which does not contain terms related to time variables. In Padfield's study, she considered the effect of the unwinding angle and of various spool shapes, including cylindrical and conical packages for outer winding and hollow packages for inner winding. Also, the initial tensile force was predicted, followed by a change in the fluid resistance coefficient. In the late 1970s, Kothari et al. [2] demonstrated that fluid resistance to the normal direction was the dominant force acting on cable, in comparison with both tangential fluid drag and gravity. This conclusion was in good agreement with Padfield's equation of motion, which assumed that the motion of the cable is only dominantly dependent on the normal fluid drag. Kothari et al. [3,4] also analyzed the unwinding motion by changing of the cone angle in cylindrical and conical packages. In the early 1990s, Fraser et al. presented mathematical evidence that the unwinding equation can be transformed to the steady-state equation using the perturbation method [5,6]. He derived the equation of motion defined in the cylindrical coordinates system and showed that the

* Corresponding author. Tel.: +82 51 510 2328; fax: +82 51 581 8514.
E-mail addresses: cae99110@pusan.ac.kr (K.-W. Kim), wsyoo@pusan.ac.kr (W.-S. Yoo).
[1] Tel.: +82 51 510 1457; fax: +82 51 512 9835.

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unwinding motions coincided with that defined in the orthogonal coordinate system. In 2013, Kim et al. [7] derived the steady-state equation of motion from Hamilton's principle for an open system and numerically verified that the perturbation method can be used to derive the steady-state equation of motion from the transient-state equation of motion.

Although the analysis of the unwinding motion has progressed continuously for many years, it has not overcome the fact that the solutions have only been calculated from the steady state. In other words, the shooting method used to solve the steady-state equation does not contain all the boundary conditions at the lift-off point, which means that a unique solution has not yet been found. Lee et al. [8,9] attempted to address this issue in 2011, by numerically solving the transient equation of motion, in which all the two-point boundary conditions are adopted exactly considering the unwinding velocity induced at the guide-eyelet point, instead of the steady-state equation of motion. That was an example of innovative research in the field of unwinding dynamics. In 2012, Kim et al. [10] derived a transient-state equation of motion containing the effects of gravity and tangential air resistance and asserted that the gravity and the tangential air resistance can be neglected in this equation of motion. However, there were limitations when applying the results to an analysis of the unwinding motion of a wire-guided cable that includes flexural rigidity. That is because Lee's results are applicable for yarn and tether, which have no flexural rigidity. Therefore, in this paper, the unwinding motion, including flexural rigidity, of a wire-guided cable is analyzed in two fluid conditions, i.e., in air and water.

This paper is structured as follows: Section 2 explains the total system of an unwinding cable and the derivation process of the unwinding equation using the expanded Hamilton's principle. In addition, the equation for the tensile force is derived from various assumptions that are induced from the inextensible condition and rules. Section 3 presents the method for solving the unwinding equation and applying the finite-difference approximation. The effect of flexural rigidity is represented by a geometrical constraint using the approximated equations when the unwinding velocity is equal to zero, and the results for a set of unwinding cables are demonstrated with various initial tensile forces in two fluid conditions (air and water). Finally, a general summary and conclusions are given in Section 4.

2. Equations for unwinding cable by taking into consideration the effect of flexural rigidity

2.1. System of unwinding cable

A system is developed to demonstrate the unwinding motion of a cable from the spool package, taking into account the effect of flexural rigidity (see Fig. 1). In this paper, the cable is assumed to be inextensible and of uniform density. $\hat{\mathbf{e}}_r$, $\hat{\mathbf{e}}_\theta$, $\hat{\mathbf{e}}_z$ are defined in the cylindrical coordinate system as the unit vectors along each direction of the coordinates. The spool package is regarded as a cylinder having radius C_r in which the cable is assumed to be initially wound. Point O is called the guide-eyelet point where the cable is pulled out with constant unwinding velocity V and point L is called the lift-off point, where the cable is withdrawn from the spool package. The control volume height is defined as H_B. P is an arbitrary point on the cable and $\vec{R}$ is a vector heading to point P that can be represented as the following equation:

$$\vec{R} = r\hat{\mathbf{e}}_r + z\hat{\mathbf{e}}_z \tag{1}$$

$\vec{\omega}$ is the angular velocity of the unwinding cable that is proportional to the unwinding velocity V and inversely proportional to the package radius C_r which coincides with the axis of the package. This means that the cable is constantly unwound from the changing lift-off point by translational unwinding velocity. Also, as the cable is assumed to be linearly elastic, homogeneous, and isotropic material, the modulus of elasticity, E, and the moment of inertia, I, can be utilized to express the effect of flexural rigidity on the cable.

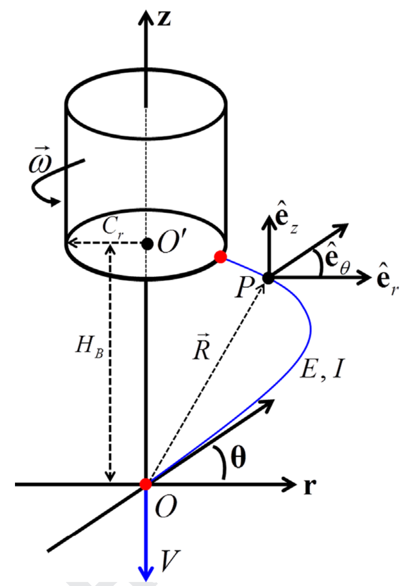

Fig. 1. System of an unwinding cable from a spool package.

2.2. Hamilton's principle for changing mass

As mentioned above, the total mass of an unwinding cable varies continuously because of the changing shape of the balloon in the control volume. Therefore, the general Hamilton's principle for a closed system can no longer be applied. Nevertheless, Hamilton's principle for a closed system is briefly introduced here to extend it to the open system.

In general, the Hamiltonian principle for a closed system in which there is no mass transport within a moving region of space $R_c(t)$ bounded by the surface $B_c(t)$ may be represented as the following equation:

$$\delta(L)_c + \delta W - \frac{D}{Dt}\int\int\int_{R_c(t)} \rho_p(\vec{u}_p \cdot \delta\vec{r}_p)\, dv = 0 \tag{2}$$

where $(L)_c = (K-E)_c$ is the Lagrangian in which $(K)_c$ is the kinetic energy and $(E)_c$ is the potential energy within the closed region $R_c(t)$ at any instant. δW is the virtual work produced by external forces and moments. The particle density is ρ_p and the velocity is $\vec{u}_p$ at position $\vec{r}_p$ at time t and $D(\,)/Dt$ is the material derivative. Eq. (2) can be transformed into Eq. (3) when the space does not move, which indicates that the surface $B_c(t)$ is always fixed in a closed system:

$$\delta(L)_c + \delta W - \frac{d}{dt}\int\int\int_{R_c(t)} \rho_p(\vec{u}_p \cdot \delta\vec{r}_p)\, dv = 0 \tag{3}$$

In other words, there is no variation in the space but there is a variation in the time in a closed system. Therefore, Hamilton's principle is obtained as Eq. (4) by integrating Eq. (3) with respect to time between two instants t_1 and t_2. Namely, $\delta\vec{r}$ vanishes at $t=t_1$ and $t=t_2$

$$\delta\int_{t_1}^{t_2} (L)_c\, dt + \int_{t_1}^{t_2} \delta W\, dt = 0 \tag{4}$$

To extend the closed system to a system with changing mass, McIver [11] introduced a time-varying system that has open

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