ARTICLE IN PRESS

International Journal of Non-Linear Mechanics ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Modified Adomian Decomposition Method for Van der Pol equations

Q1 P.V. Ramana, B.K. Raghu Prasad

Q3 *Department of Civil Engineering, Indian Institute of Science, Bangalore 560 012, India*

ARTICLE INFO

Article history:
Received 11 April 2011
Received in revised form
28 February 2014
Accepted 12 March 2014

Keywords:
Parabolic problems
Initial value problems
Modified Adomian Decomposition Method
Analytical solution

ABSTRACT

In the paper, the well known Adomian Decomposition Method (ADM) is modified to solve the parabolic equations. The present method is quite different than the numerical method. The results are compared with the existing exact or analytical method. The already known existing Adomian Decomposition Method is modified to improve the accuracy and convergence. Thus, the modified method is named as Modified Adomian Decomposition Method (MADM). The Modified Adomian Decomposition Method results are found to converge very quickly and are more accurate compared to ADM and numerical methods. MADM is quite efficient and is practically well suited for use in these problems. Several examples are given to check the reliability of the present method. Modified Adomian Decomposition Method is a non-numerical method which can be adapted for solving parabolic equations. In the current paper, the principle of the decomposition method is described, and its advantages are shown in the form of parabolic equations.

1. Introduction to modified adomian decomposition method [MADM]

Current developments in mathematical physics and engineering problems have given impetus to research on non-linear partial differential equations (NLPDE) and linearization techniques. Unfortunately, such techniques which assume essentially that a non-linear system is almost linear often have little physical justification. It has become vital, not only to theory but also to the areas of practical application, that further advances be made. In recent years the development of the high-speed digital computer and increased interest in non-linear phenomena have led to an intensive study of the numerical solution of ordinary and partial differential equations (O & PDEs). Adomian Decomposition Method (ADM), developed by George Adomian [7,2] is one such. The ADM is a non-numerical method for solving non-linear differential equations, both ordinary and partial. The general direction of this work is towards obtaining solution for Partial Differential Equations (PDEs).

In the 1985s, Adomian [7,2] proposed a new and ingenious method to obtain exact solution of linear and non-linear equations of various kinds like algebraic, differential for both ordinary and partial, integral, etc. problems. The technique uses a decomposition of the non-linear operator as a series of Adomian functions. Each term of this series is a generalized polynomial called the Adomian polynomial. Some techniques which assume essentially that the non-linear system is almost linear after equivalent linearization will not be able to retain the originality of the problem. ADM consists of splitting the given equation into linear, remainder and non-linear parts, inverting the highest order differential operator contained in the linear operator on both sides, identifying the initial or boundary conditions and the terms involving the independent variable alone as initial approximation, decomposing the unknown function into a series whose components are to be determined, decomposing the non-linear function in terms of special polynomials, and finding the successive terms of the series solution. The ADM provides the solution in a rapidly convergent series with easily computable components. The main advantage of the method is that it can be used directly to solve, all types of differential equations with homogeneous and inhomogeneous boundary conditions. Another advantage of the method is that it reduces the computational work in a tangible manner, while maintaining higher accuracy of the numerical solution. The ADM can be shown to solve effectively, easily and accurately a large class of linear and non-linear, ordinary, partial, deterministic or stochastic differential equations with closed form solutions, which converge rapidly to accurate solutions. It is shown that the ADM is more efficient than many other numerical methods [1,12].

The ADM consists of calculating the solutions of non-linear functional equations using infinite Taylor's series in which each term can be easily determined. ADM is used to obtain the nth-order approximation to the one direction partial solution that satisfies the boundary conditions in the same direction. The decomposition solution is also an approximation, but one which does not change the originality of the problem. Therefore it is often physically more realistic. The solution obtained by decomposition is generally an infinite series. Commonly exact methods are used for solving non-linear equations. Some times numerical methods are used in special cases, where exact methods cannot be

E-mail address: pvramana@civil.iisc.ernet.in (P.V. Ramana).

http://dx.doi.org/10.1016/j.ijnonlinmec.2014.03.006

used to solve equations of higher order non-linear problems. But numerical methods often suffer problem due to convergence. Numerical techniques, which are commonly used, encounter difficulties in terms of size step truncation and round off errors causing a loss of accuracy.

The ADM transforms the non-linear equation into a recursive relation. The ADM has proved to be very effective in saving the computational effort. An application in seismic analysis can reduce the computational time by an order of magnitude.

The novel tool derived by Adomian [3–6] has been applied to a wide class of ordinary and partial differential equations. However as mentioned earlier the ADM has also several limitations in terms of accuracy and speed. Therefore in the current paper the ADM has been modified to address the lacunae mentioned earlier. The modified method is named as Modified Adomian Decomposition Method or in short MADM. In the following papers the same has been described in detail. MADM has been applied to solve many functional equations. MADM has a useful features in that it provides the solution in a rapid convergent power series with elegantly computable series of terms. The MADM has proved to be very effective and results in considerable savings in computation time as well as accuracy.

2. Modified adomian decomposition method procedure

Consider a dynamical system $LN(u,\acute{u},\ldots)=g(x,y,z,t)$, where LN is a differential operator. $LN(LN(u)=Lu+Ru+Nu)$ has to be that of the highest derivative both in ADM and MADM, if the problem is a one-dimensional (governed by ODE) with only space or time as the independent variable. If it is two-dimensional like plate problems $(x,y,$ or $t)$ or one-dimensional time variant $(x,t$ or $y,t)$ or three-dimensional like thick plate $(x,y,z$ and $t)$, when we come across partial differential equation, MADM is more advantage than ADM. For problems like the above with time also as an independent variable, it is only in MADM one comes across with only one derivative operator, which is the highest order either w.r.t. x or w.r.t. y or w.r.t. z or w.r.t. time. Here $LN(u)=Lu+Ru+Nu=g(t)$ includes both linear and non-linear terms. The linear term is written as $Lu+Ru$ where L is chosen as the higher ordered derivative, and R contains the remainder terms. The MADM consists of the following:

1. The given equation $L(u,\acute{u}\ldots)u+R(u,\acute{u}\ldots)u+N(u,\acute{u}\ldots)u=g(t)$ is split into linear (L), remainder (R), and non-linear (N) parts where L is any higher order linear derivative operator, N is the non-linear operator and R denotes the remainder terms, other than linear and non-linear derivative operators.
   Here an important contribution to be highlighted is that in ADM L is chosen as the highest order derivative, while in MADM it is any higher ordered derivative chosen according to initial conditions.
2. Selecting any one of the spatial or time $(x,y,z$ or t, preferably time variable for initial value problem) variables and considering the higher order derivative w.r.t. that variable, higher order linear derivative operator L is retained on the L.H.S, while remaining (other) terms viz. N and R are transferred to R.H.S. Now the given equation becomes $Lu=g-Ru-Nu$. Inverting the higher order linear operator on both the sides, the equation becomes $L^{-1}Lu=L^{-1}g-L^{-1}Ru-L^{-1}Nu$.
3. On successive integrations $u=a_0+t\,a_1+t^2\,a_2+\cdots+t^n a_n+L^{-1}g-L^{-1}Ru-L^{-1}Nu$, where $a_0,a_1,\ldots a_n$ are n integral constants.
4. Identifying the initial conditions and the terms involving the independent variable alone as initial approximation $u_i=a_0+t\,a_1+t^2\,a_2+\ldots+t^n a_n+L^{-1}g$.
5. Having obtained u_i as stated above, the various u_i, $i=0$ to n are obtained as follows. Decomposing the unknown function into a series whose components are to be determined, the decomposed terms are

   $$u_{00}=a_0,$$
   $$u_0=t\,a_1+L^{-1}g$$

   where x_i's are known independent variables. The integral constants $a_0,\ldots a_n$ are determined from the given boundary conditions.
6. Finding the successive terms of the series solution by successive iteration using Adomian polynomials, in the next recursive steps, we obtain

   $$\begin{aligned}
   u_1&=a_2\,x_i^2-L^{-1}Ru_0-L^{-1}Nu_0=a_2\,x_i^2-L^{-1}Ru_0-L^{-1}A_0,\\
   u_2&=a_3\,x_i^3-L^{-1}Ru_1-L^{-1}Nu_1=a_3\,x_i^3-L^{-1}Ru_1-L^{-1}A_1,\\
   &\vdots\quad \ldots\vdots\quad \ldots\\
   u_n&=a_{n+1}\,x_i^{n+1}-L^{-1}Ru_{n-1}-L^{-1}Nu_{n-1}\\
   &=a_{n+1}\,x_i^{n+1}-L^{-1}Ru_{n-1}-L^{-1}A_{n-1},
   \end{aligned}$$

   the non-linear terms $Nu_0,\ldots Nu_n$ are further decomposed with the help of Adomian polynomials as

   $$\begin{aligned}
   Nu_0&=A_0,\\
   Nu_1&=A_1,\\
   &\vdots\\
   Nu_n&=A_n.
   \end{aligned}$$

In an Initial Value Problem (IVP), L is d^n/dt^n (a differential operator) and L^{-1} may be regarded as multiple definite integrations with limits 0–t. For IVPs, they need to be found from initial conditions and may be identified as $u|_{t=0}$ and $du/dt|_{x=0}$. In the MADM for ith time interval, L_t^{-1} is the two-fold definite integral from 0 to δT, with $\delta T\in[t_{i-1},t_i]$. The same procedure for singularity, fractional derivative and fracture problems have been adopted. The subset of time interval $[0,T]$ be divided into n subintervals and so ordered that $0=t_0<t_1<t_2<\cdots<t_n=T$ and $\delta T=t_i-t_{i-1}$. Therefore, using MADM over ith interval, the solution similar to numerical approximation procedure. To convey the idea and for the sake of completeness of the MADM, now one can rewrite the non-linear equation in the following form of

$$u=N(u), \tag{1}$$

It is convenient to find the solution of equation (1) in the series form such as

$$u=\sum_{i=0}^{\infty}u_i. \tag{2}$$

The non-linear operator N is decomposed as shown below:

$$N\left(\sum_{i=0}^{\infty}u_i\right)=N(u_0)+\sum_{i=0}^{\infty}\left[N\left(\sum_{j=0}^{i}u_j\right)-N\left(\sum_{j=0}^{i-1}u_j\right)\right]. \tag{3}$$

Another important contribution towards modifying the ADM is choosing the term containing the independent variable such as $a_i t^n$ only one at a time. By doing so, the computation is simplified enormously particularly when non-linear terms in Nu are involved. In ADM the non-linear terms containing higher powers of u will result in higher powers of a sum of several terms of Adomian polynomial. Expanding such sum raised to higher powers is not only tedious but also leads to errors. As mentioned above such steps are avoided in MADM by taking terms one by one.

Some further comments about MADM: A damped non-homogeneous NLODE model representing a dynamical behavior of mechanical systems. In analytical techniques, it becomes necessary to

Please cite this article as: P.V. Ramana, B.K. Raghu Prasad, Modified Adomian Decomposition Method for Van der Pol equations, International Journal of Non-Linear Mechanics (2014), http://dx.doi.org/10.1016/j.ijnonlinmec.2014.03.006