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Effect of non-linear damping on the structural dynamics of flapping beams

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ABSTRACT

In this paper we investigate, through experiment and simulation, the effects of non-linear damping forces on the large amplitude structural dynamics of slender cantilever beams undergoing flapping motion in air. The aluminum beams are set into flapping motion through actuation at the beam base via a 4-bar crank-and-rocker mechanism. The beam strain response dynamics are investigated for two flapping amplitudes, 15° and 30°, and a range of flapping frequencies up to 1.3 times the first modal frequency. In addition to flapping at standard air pressure, flapping simulations and experiments are also performed at reduced air pressure (70% vacuum). In the simulations, linear and non-linear, internal and external damping force models in different functional forms are incorporated into a non-linear, inextensible beam theory. The external non-linear damping models are assumed to depend, parametrically, on ambient air density, beam width, and an empirically determined constant. Periodic solutions to the model equation are obtained numerically with a 1-mode Galerkin method and a high order time-spectral scheme. The effect of different damping forces on the stability of the computed periodic solutions is analyzed with the aid of Floquet theory. The strain-frequency response curves obtained with the various damping models suggest that, when compared to the linear viscous and non-linear internal damping models, the non-linear external damping models better represent the experimental damping forces in regions of primary and secondary resonances. In addition to providing improved correlation with experimental strain response amplitudes over the tested range of flapping frequencies, the non-linear (external) damping models yield stable periodic solutions for each flapping frequency which is consistent with the experimental observations. Changes in both the experimental ambient pressure and flapping amplitude are determined to result in some variation in the non-dimensional parameters associated with each of the non-linear external damping models. This result likely indicates an incomplete description of the model parameter dependence and/or non-linear functional form of the damping force.

1. Introduction

The characterization of the large amplitude vibration of actuated, slender beam structures has been, and still is, important for many engineering applications including developing technologies such as flapping-wing micro aerial vehicles (MAVs) [1,2], biomimetic robotic propulsion [3,4], electronic cooling devices [5,6], and energy harvesting mechanisms [7,8]. While the simple geometry of beams would appear to make their response characterization somewhat simple, when the amplitude of vibration becomes comparable to its length, various effects including geometric, inertial, and damping non-linearities complicate the analysis. Motivated by the results of a previous study by the authors which is described in more detail below, in the present study we are interested in gaining a better understanding of the non-linear damping mechanism acting in large amplitude beam vibrations.

In reference [9] the authors studied, through simulation and experiment, the non-linear structural dynamics of an aluminum beam actuated at its base through the action of a 4-bar crank-and-rocker mechanism. Two different flapping (actuation) amplitudes were investigated for a number of frequencies ranging up to, and slightly beyond, the first modal frequency of the beam. The numerical model used in the simulation was based upon a geometrically non-linear beam finite element model and a finite-difference based time-marching scheme. While overall the simulation and experiment compared favorably, in the regions of primary and secondary (superharmonic) resonance the simulation significantly overestimated the experimental response. In addition, while the simulation predicted a number of types of trajectories ranging from periodic to irregular (possibly chaotic), all

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experimental trajectories were found to be periodic. While a number of explanations were put forth for these discrepancies, the authors believe that the main inadequacy of the mathematical model used in reference [9] was the use of a linear viscous damping model (mass-proportional Rayleigh damping) with a coefficient determined by using small amplitude free vibration tests.

Dissipation of mechanical energy in vibrating structures is most often referred to as damping and is related to a number of different mechanisms which operate inside (internal) or outside (external) of the structure. Internal damping (or material damping) can be associated with several mechanisms which include, to name only a few particular to metals, grain boundary viscosity, point defect relaxations, intercrystalline thermal currents, dislocation mechanisms, and localized plastic deformation [10,11]. In general, damping forces which arise from external mechanisms are larger than those which are due to internal mechanisms. These external damping mechanisms may include dry friction at the structure's contact joint and various forms of fluid–structure interactions governed by the viscous, inertial, and convective forms of momentum transport which take place between the structure and the surrounding fluid medium [12].

The fluid forces acting on a bluff body, a cylinder for instance, which undergoes oscillatory motion in an incompressible viscous fluid have been approximated for decades based upon a semi-empirical approach proposed by Morison et al. [13]. According to the Morison model, the oscillatory fluid force exerted on the body is regarded as being contributed by two components termed “added mass” and “fluid damping” which are in-phase and out-of-phase with the acceleration of the body, respectively [13–15]. These force components are expressed as velocity-squared-dependent drag force and acceleration-dependent inertial force with the coefficients determined experimentally [16]. The added mass component is known to be responsible for lowering the *in vacuo* resonance frequencies of the structure while the fluid damping component is the primary cause of the dissipation of the structure's mechanical energy. The added mass (or virtual mass) force is due to the acceleration imparted on the mass of the fluid displaced by the body. On the other hand flow separation in viscous fluids produces vortices with out-of-phase transport velocities which in turn give rise to vortex-shedding-induced fluid damping forces on the body [15,17].

When a body with salient edges is moved through a placid fluid, the flow separation occurs almost immediately after the motion begins [18]. In order to model the separated flow around a *rigid* flat plate with sharp edges, and to determine the fluid forces acting on the plate, Jones [19] derived ordinary differential equations governing the evolution of the velocity field using a boundary integral formulation and an inviscid flow assumption. The motion of the plate, which is assumed to be normal to the quiescent inviscid fluid, gives rise to a two dimensional flow field composed of a bound vortex sheet on the plate surface and free vortex sheets emanating from both edges. Inspired by the movements of flapping insect wings, Jones [19] numerically investigated the fluid vortex patterns and pressure forces induced by the unsteady motion of the flat plate during its deceleration, stopping, and re-acceleration in the reverse direction. It was determined that, during motion reversal of the plate, new starting vortices form and merge into the stopping vortices, resulting in a highly non-linear fluid forcing regime.

In the case of a slender flexible beam executing large amplitude oscillations, the mathematical modeling of damping forces exerted on the beam structure by the surrounding quiescent fluid is a much more difficult task. The damping forces acting on the structure are strongly coupled with the structural motion and have non-linear dependence on both the amplitude and frequency of the structural oscillations [17,20]. Recently, Bidkar et al. [17] combined an inviscid vortex-shedding fluid model of Jones [19] and a linear Euler–Bernoulli beam model to develop a fluid–structure interaction model for predicting the non-linear aerodynamic damping force acting on piezoelectrically excited cantilever beams oscillating with large amplitudes compared to their widths. The model is based upon a small deflection, single harmonic response assumption and requires experimentally measured *in vacuo* mode shape, frequency, and amplitude in order to capture large deflection effects. Despite the slight overestimation of the aerodynamic damping force, the semi-empirical model utilized in this work gives better predictions when compared to previous studies which were based on purely inviscid or purely viscous diffusion theories [21].

In recent studies, Aureli et al. [12,22] improved the complex hydrodynamic function approach of Sader [21] to take into account the effect of vortex shedding and added mass on the non-linear fluid damping loads experienced by the cantilever beams undergoing large amplitude oscillations. They concluded that the proposed theoretical and numerical framework is generally able to accurately predict the resonance frequencies and damping factors. Kopman and Porfiri [23] combined Morison's fluid force model with the Euler–Bernoulli beam model in an effort to predict the thrust force produced by the flexible caudal fin of a robotic fish. The Morison model coefficients were determined empirically for three different fin geometries and a range of tail-beating frequencies (1–2 Hz) and amplitudes (10–20°). The model prediction agreed well with the experimental thrust data in the studied range of input parameters. In their piezohydroelastic model, Cha et al. [24] utilized the Morison formula to simulate the damping effect of the encompassing water medium on the piezoelectric energy harvesting efficiency of slender, base-excited cantilever beams. Model results were found to corroborate the experimental results for a number of submersion lengths.

In the present research our primary objective, which is motivated by our previous findings [9], is to investigate the effect of non-linear damping forces on the structural dynamics of slender cantilever beams undergoing flapping motion with amplitudes much larger than their width. In order to achieve this objective, we use both experiment and simulation. The simulation is conducted using a theoretical model which incorporates simple non-linear damping models in various functional forms containing an empirically determined parameter into a non-linear inextensible beam theory. Such simple analytical models for damping are used to compensate for the inability, or unwillingness, to solve the true (complex) fluid–structure interaction problem [20]. Such an approach is widely used in the literature [25–29], and if the parameters are chosen correctly it yields an analysis framework which can accurately and efficiently predict large amplitude beam vibration response. The numerical solution consists of a 1-mode Galerkin method for spatial discretization and a high-order pseudospectral/collocation method for temporal discretization. In addition, to explore the effect of damping on the stability of periodic solutions, Floquet theory is used in conjunction with the numerical solutions. The experimental setup which is used is the same as that presented in reference [9], with the addition of a vacuum chamber.

The remainder of the paper is organized in the following manner. In Section 2 the experimental apparatus, which primarily consists of a 4-bar mechanism for beam actuation and a vacuum chamber, is summarized along with the experimental procedure. In Section 3 we present the non-linear, inextensible beam model and the time-dependent boundary conditions used to approximate the experimental 4-bar mechanism actuation. Approximate solution of the problem in the spatial and time domains is then presented in Section 4, and the linear and non-linear damping