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Vibro-impact dynamics of a two-degree-of freedom periodically-forced system with a clearance: Diversity and parameter matching of periodic-impact motions

G.W. Luo *, X.H. Lv, Y.Q. Shi

Key Laboratory of System Dynamics and Reliability of Rail Transport Equipment of Gansu Province, Lanzhou Jiaotong University, Lanzhou 730070, PR China

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ABSTRACT

A two-degree-of-freedom system with a clearance and subjected to harmonic excitation is considered. The correlative relationship and matching law between dynamic performance and system parameters are studied by multi-parameter and multi-performance co-simulation analysis. Two key parameters of the system, the exciting frequency ω and clearance δ, are emphasized to reveal the influence of the main factors on dynamic performance of the system. Diversity and evolution of periodic impact motions are analyzed. The fundamental group of impact motions is defined, which have the period of exciting force and differ by the numbers p and q of impacts occurring at the left and right constraints of the clearance. The occurrence mechanism of chattering-impact vibration of the system is studied. As the clearance δ is small or small enough, the transition from 1–p–p to 1–$(p+1)$–$(p+1)$ motion (the fundamental group of motions, $p \geq 1$) basically goes through the processes as follows: pitchfork bifurcation of symmetric 1–p–p motion, period-doubling bifurcation of asymmetric 1–p–p motion, non-periodic or chaotic motions caused by a succession of period-doubling bifurcations, symmetric 1–$(p+1)$–$(p+1)$ motion generated by a degeneration of chaos. As for slightly large clearance, a series of grazing bifurcations of periodic symmetrical impact motions occur with decreasing the exciting frequency so that the number p of impacts of the fundamental group of motions increases two by two. As p becomes big enough, the incomplete chattering-impact motion will appear which exhibits a chattering sequence in an excitation period followed by a finite sequence of impacts with successively reduced velocity and reaches the non-sticking region. Finally, the complete chattering-impact motion with sticking will occur with decreasing the exciting frequency ω up to the sliding bifurcation boundary. A series of singular points on the boundaries between existence regions of any adjacent symmetrical impact motions with fundamental period are found, i.e., two different saddle-node bifurcation boundaries of one of them, real-grazing and bare-grazing bifurcation boundaries of the other alternately and mutually cross themselves at the points of intersection and create inevitably two types of transition regions: narrow hysteresis and small tongue-shaped regions. A series of zones of regular periodic and subharmonic impact motions are found to exist in the tongue-shaped regions. Based on the sampling ranges of parameters, the influence of dynamic parameters on impact velocities, existence regions and correlative distribution of different types of periodic-impact motions of the system is emphatically analyzed.

1. Introduction

The clearances or constraints among mechanical components exist inevitably in mechanical systems due to the restriction on technological level in the mechanical machining process, the demand for mechanical design and assembly errors, the need for the heat-expansion and cold-contraction of some part of mechanical components or other reasons. The existence of the clearances or motion limiting constraints is bound to bring about the vibro-impact effects which have a great influence on performance of mechanical equipments, increase noise levels and intensify fatigue and abrasion of mechanical parts. The non-smooth natures of phase trajectories caused by the impacts complicate the dynamic analysis of such systems inevitably, but they can be described theoretically and numerically by the discontinuities in good agreement with reality. It is necessary to be able accurately to model the dynamics of the mechanical systems with clearances or motion limiting constraints from the multi-parameter level, so as to utilize effectively and enlarge profitable effects by reasonable

* Corresponding author.
E-mail address: luogw@mail.lzjtu.cn (G.W. Luo).

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parameter matching design, to furthest minimize the influence of unfavorable factors such as abrasion, chattering-impact, high noise levels, etc. The broad interest in revealing dynamical performance of vibro-impact systems is reflected by an ever increasing amount of investigations devoted to this aspect. Several effective methods of qualitative, numerical and experimental analyses have been proposed and their validity has been demonstrated by different vibro-impact models in the past several years. Dynamics of different types of vibratory systems with clearances or motion limiting constraints, including detailed observations about stability and bifurcations using numerical simulations and qualitative analyses, were partly reported in Refs. [1–10]. Shaw and Holmes [1] studied a single-degree-of-freedom vibro-impact system by casting the traditional approaches used for analyzing the periodic responses in the system in the form of a more general Poincaré map. The notable work showed how even a very simple impact oscillator could display the full range of typical nonlinear behavior: saddle node and flip bifurcations, multiple coexisting attracting solutions, chaos, etc. Periodic and chaotic behavior of a threshold-limited two-degree-of-freedom system was analyzed by Aidanpää and Gupta [2]. Souza and Caldas [3] put forward an algorithm of the spectrum of Lyapunov exponents of attractors of a vibro-impact system for interpreting bifurcations and chaotic transitions. Peterka and Kotera [4] studied the appearance and characteristics of intermittency chaos of an impact oscillator occurring near saddle-node bifurcation boundaries of neighboring fundamental motions with two different impacts in an excitation period. Pavlovskaia and Wiercigroch [5,6] developed an efficient semi-analytical method to study the percussive drilling principle for the impact systems with drift and accurately predicted a range of control parameters for which the best progression rates were obtained. Quinn and Bairavarasu [7] considered a three-mass collision model with compliant contacts, a two-dimensional map is derived to describe initial conditions that lead to pairwise sequences, and the variation in the final velocities is characterized in terms of the initial configuration of the system. Luo [8] determined analytically and numerically stability and bifurcation conditions for the LR model motion in an impact oscillator, for which the regions are developed in parameter space. Leine [9] presented an asymptotic approximation method for the critical restitution coefficient of a parametrically excited impact oscillator and described its dynamics by an unilaterally constrained Hill's equation. More studies on vibro-impact dynamics were summarized in Ibrahim's monograph [10]. A special feature of vibro-impact systems that might prove to be useful in the current study is the instability caused by low-velocity collisions, which is the so-called grazing effect. The pioneer work in this field was done by Nordmark [11], who developed systematic methods for investigating grazing dynamics and their attendant bifurcations, providing the results which laid the foundation for many subsequent studies. Thereafter, this work has been further expanded by discontinuity-geometry approaches of a series of impact maps, where the increasingly thorough investigations of grazing bifurcations of such maps have been reported in Refs. [12–25]. The chattering-impact vibration denotes an accumulation of impact events in finite time (or a motion period), and sticking is where the dynamics becomes finally constrained to the impact surface as the impact attenuation ends. Detailed studies of chattering-impact vibration with sticking were carried out for the vibro-impact systems in Ref. [26–35]. Rising events occurring in multi-sticking solutions are discussed by Wagg [29,30], the study on simulation and stability analysis of impacting systems with complete chattering has been further expanded by Nordmark and Piiroinen [31]. de Souza and Caldas [32] numerically investigated the basins of attraction of coexisting periodic and chaotic attractors in a gear-rattling impact model. Luo [33,34] presented an idealized, piecewise linear system to model non-smooth vibration of gear transmission arising from impacts between the gear teeth and predicted analytically occurrence of the periodic impact-sticking motions. Hősa and Champneysb [35] considered a mechanical model of a pressure relief valve and analyzed the geometry of the chattering region via the computation of several pre-images of the grazing set. The vibro-impact systems display many of the complex dynamical features near critical points of high codimension bifurcations; in particular, different types of codimension two bifurcations and parameter unfolding associated with the interaction of saddle-node and grazing bifurcations [23], double flip bifurcation [36], Hopf-flip bifurcation [37], Hopf-pitchfork bifurcation and double Neimark–Sacker bifurcation [38], codimension-two grazing bifurcations [39,40], grazing-sliding bifurcation [41], etc., were studied by numerical simulations and qualitative analyses in recent years. Several methods of experimental analyses have been developed for some impact models in Refs. [42–46]. Wiercigroch and Sin [42] performed an experimental study of base excited symmetrically piecewise linear oscillator for observing and proving symmetric and asymmetric periodic-impact motions and their attendant bifurcations. Jin and Hu [43] designed an experimental setup consisting of both vibro-impacting cantilever beams with lumped mass blocks for testing various types of vibro-impact motions such as subharmonic, superharmonic, quasi-periodic and chaotic motions, etc. Quinn [44] considered the effect of external forces during finite duration collisions using an incremental model of impact. Based on the results of the experimental system and the incremental model, the standard algebraic model of restitution is modified to include these finite duration effects. Nguyen et al. [45] designed an experimental scheme of vibro-impact rig to mimic a mole penetration through the soil, which was based on electro-mechanical interactions of a conductor with an oscillating magnetic field. Sitnikova and Pavlovskaia [46] studied bifurcations and chaotic attractors of an impact oscillator with a shape memory alloy constraint. The evolution of chaotic attractors is recorded experimentally, and changes in the structure of the attractors are shown. Along with the requirement of engineering application and the increasing of basic research on non-smooth dynamics, vibro-impact dynamics has been applied to a wide range of practical mechanical systems for finding the correlative relationship between dynamic performance and model parameters, e.g., wheel–rail impacts of railway coaches [47,48], vibrating hammer [49], pile driver [50,51], ground moling dynamics [52], mill rolls [53], the fly-wheel model of the bouncing ball [54], link mechanism [55], ultrasonic percussive drilling [56,57], the vibro-impact capsule system [58], Jeffcott rotors with bearing clearance [59–61], impact dampers [62,63], excited pendula with impacts [64], high frequency vibro-impact drilling [65], pipes conveying [66], vibro-impact interaction of ships with ice [67], gears transmission systems [32–34,68–71], etc. The dynamical models designed in Refs. [47–71] have proved to be useful for revealing dynamical performance of such practical systems and the related studies have provided much information of the fundamental nature that broadened the scope of knowledge on the non-smooth motions of mechanical systems with impacts.

It is important to note, that most studies of vibro-impact dynamics are based on single-parameter bifurcation analysis in the past several years. Very few have considered multi-performance, multi-process coupling and multi-parameter simulation analysis and collaborative optimization for research on vibro-impact dynamics. The purpose of the present study is to focus attention on multi-performance and multi-parameter simulation analysis for dynamics of a two-degree-of-freedom periodically-forced system with a clearance represented by two symmetric rigid stops. The correlative relationship between dynamic performance and system parameters is particularly studied to find their

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