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Response and instability prediction of helicopter dynamics on the ground

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ABSTRACT

In helicopters with hinged blades an unstable dynamical phenomenon known as ground resonance may occur during take-off and landing and lead to the total destruction of the aircraft. Predicting the phenomenon is necessary to determine the stability of periodical equations of motion. The instability boundaries can be easily obtained for isotropic rotor configurations through multi-blade coordinate transformation once the periodic terms are eliminated. However, Floquet's theory is commonly used to treat the periodic motion equations when introducing the asymmetric effects of spring or damper aging or rotor rupture (anisotropic rotors). In additional, it is known that when treated as parametric excitations, periodic terms may lead to instability in dynamical systems under parametric resonances. In this paper a helicopter in contact with the ground is considered as a parametrically excited system and the equations are treated analytically by applying the method of multiple scales (MMS). A stability analysis verifies the existence of parametric instabilities by first order sets of equations for an isotropic rotor configuration. The results are compared and validated with those obtained by using Floquet's Theory. Moreover, the amplitude responses of the aircraft at equilibrium in the remaining resonant cases are studied. The results are then compared with those obtained from the time response analysis.

1. Introduction

An unstable phenomenon in helicopter dynamics known as ground resonance consists of a self-excited oscillation caused by the interaction between rotor blade oscillations and other forms of movement in the helicopter [1]. Several accidents have been recorded that have shown that, under certain conditions, the phenomenon can be very violent and lead to the total destruction of the aircraft.

The earliest research into the phenomenon was performed by Coleman and Feingold [2] who laid the foundations for all subsequent studies into the problem. Donham et al. [3] and Lytwyn et al. [4] added the air resonance effect and verified its influence on the phenomenon. Major contributions towards understanding the phenomenon of ground resonance in hingeless and bearingless rotors were made by army researchers, such as Hodges [5]. Recently, Kunz [6] analyzed the influence of non-linear springs and dampers (elastomeric elements) for predicting the rotor instability zone while Byers and Gandhi [7] explored how the problem might be controlled passively.

The investigations mentioned above demonstrated that the occurrences of ground resonance can be accurately predicted for articulated, hingeless and bearingless rotors. The use of linearized equations of motion provides very accurate frequency prediction.

These works all focused on analyzing isotropic rotor configurations (all blades having the same properties). The boundary speeds of ground resonance are easily obtained once the periodic terms are eliminated through a variable transformation known as the Coleman Transformation or, more generally, as the multi-blade coordinate transformation [8].

Nevertheless, the case of anisotropic rotors is very interesting from the practical point of view. Indeed, due to aging, damper and stiffness properties can change from one blade to another. Consequently, as the Coleman Transformation can no longer be applied to such anisotropic rotors, the periodical equations of motion are subjected to a stability analysis.

In a recent study on helicopter dynamics, and more specifically on the ground resonance phenomenon, Sanches et al. [9], retained the periodic terms in the equations of motion. Using Floquet's Method the instability zones predicted were similar to those Coleman and Feingold predicted for an isotropic rotor.

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Nomenclature

Symbol	description, units
a	rotor eccentricity, m
b	blade center of giration, m
$[c.c]$	complex conjugate terms
D_n^p	$\partial^p/\partial T_n^p$ – partial derivative with respect to time scales
$\mathbf{F}_{ext}$	external force vector
$\mathbf{G}$	damping matrix of the dynamical system
i	complex number
I_{Zbk}	lag rotational inertia of the kth blade around its center of gravity, kg m^2
$\mathbf{K}$	stiffness matrix of the dynamical system
K_{bk}	kth blade lead-lag stiffness, N m rad^{-1}
K_{f_X}, K_{f_Y}	longitudinal and transversal stiffness of fuselage, N m^{-1}
m_f, m_{bk}	fuselage and mass of kth blade, kg
$\mathbf{M}$	mass matrix of the dynamical system
N_b	number of blades in the rotor
$P(\Gamma,\sigma)$	characteristic polynomial equation of 4th degree in
r_{ak}	$\sqrt{ar_{bk}}$
r_{bk}	ratio between the static moment over the total lead-lag rotational inertia of the kth blade, m^{-1}
r_{mk}	ratio between the static moment of the kth blade over the total mass of the helicopter, m
$\mathbf{S}$	state space matrix
t	time, s
T_0, T_1	time scales, s
$\mathbf{u}$	general variables
$\mathbf{V}$	state variables
$x(t)$, $y(t)$	longitudinal and transversal displacement of the fuselage, m
x_{bx}, y_{bx}	blade position in x and y directions, m
(x, y, z)	mobile coordinate system attached to the rotor hub
(X_0, Y_0, Z_0)	inertial referential system

Greek Letters

α, β	given by the relation in
ϵ	bookkeeping parameter
$\varphi_k(t)$	lead–lag angle of kth blade, rad
$\mathbf{\Phi}$	transition matrix (Floquet's Theory)
Γ	solutions of the characteristic polynomial equation
λ	characteristic exponents
Ω	rotor speed, rad s^{-1}
σ	frequency detuning parameter, rad s^{-1}
ζ_k	azimuth angle for the kth blade, rad
$\omega_{1\ldots6}$	the six general natural frequencies of Eq. (14), rad s^{-1}
ω_{bk}	lag resonance frequency of the kth blade at, rad s^{-1}
ω_x, ω_y	fuselage resonance frequencies in x and y directions, rad s^{-1}

In previous studies the anisotropic rotor was treated by using Floquet's Theory [10,11]. Since the stability analysis is performed through the monodromy matrix and calculated for each combination of input parameters, the current method is expensive in terms of computer time.

Analytical mathematical methods have already been developed and dedicated to calculating differential equations with periodical coefficients. Moreover, once the analytical responses are determined, the boundaries of instabilities can easily be obtained for any type of rotor configuration. Among these methods mention can be made of [12,13]: Hill's Infinite Determinant, Harmonic Balance Method and Method of Multiple Scales.

The latter method has been frequently applied to periodic, parametric and non-linear problems in industry [14] and in rotating dynamic systems [15–17].

Classic examples of parametrically excited systems (i.e., pendulum dynamics over a parametrically excited base and the oscillatory motion of a string under harmonic axial forces) show that the system is dynamically unstable under parametrical resonance conditions [18].

The present work therefore considers the system as a parametrically excited system and treats it by using the Method of Multiple Scales. The boundaries of the ground resonance phenomenon are obtained by a stability analysis for an isotropic rotor configuration. The results are compared with those obtained by using Floquet's Theory. Then, analysis of the amplitude response of the helicopter is carried out on the remaining parametric resonances.

In Section 2, the dynamical equations of motion from the mechanical model are formulated by neglecting the aerodynamic forces and no viscous damping is taking into account. In Sections 3 and 4, the methodologies applied to the set of periodic motion equations, i.e. the Floquet's Method (FM) and the Method of Multiple Scales (MMS), are described. In Section 5, the critical rotor speeds predicted by both methods are presented and compared for an isotropic rotor configuration. A description is also given of the development of the amplitude responses analysis and the results, while Section 6 presents the conclusions.

2. Mechanical model

The mechanical model used is similar to that proposed by Coleman and Feingold and is developed to characterize the dynamical behavior of a helicopter with a hinged rotor. In other words, it consists of figuring out the relation between the longitudinal and lateral displacement – $x(t)$ and $y(t)$ – of the fuselage and the kth blade lag angle – $\varphi(t)$ – in terms of rotor speed Ω and time t. Fig. 1 provides a general diagram of the system.

The fuselage is considered as a rigid body with its center of gravity at point O. At the outset or initial time, the origin of an inertial coordinate system (X_0, Y_0, Z_0) is coincident at this point. The body is connected to springs that represent the flexibility of the landing skid. The rotor head system consists of an assembly of one rigid rotor hub with Nb blades. Each blade is represented by a damped mass located at a distance b from the lag articulation

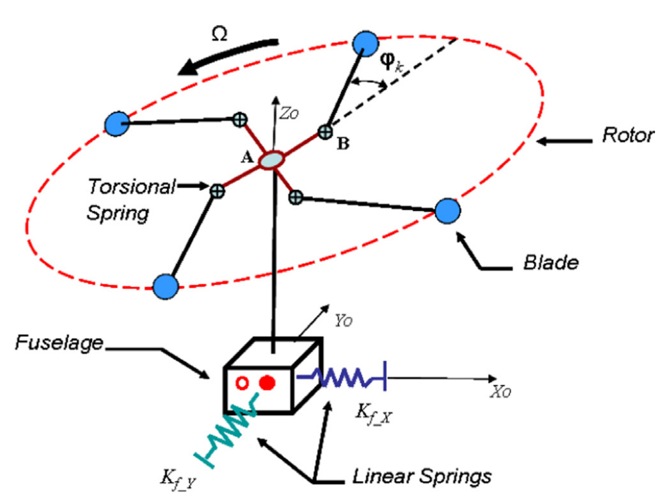

Fig. 1. Diagram of the mechanical system.

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