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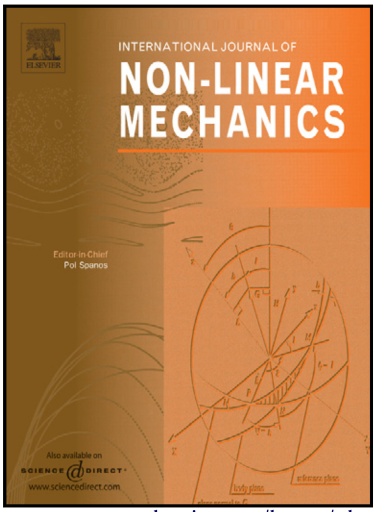

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Equilibria and Instabilities of a Slinky: Discrete Model

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The Slinky is a well-known example of a highly flexible helical spring, exhibiting large, geometrically nonlinear deformations from minimal applied forces. By considering it as a system of coils that act to resist axial, shearing, and rotational deformations, we develop a two-dimensional discretized model to predict the equilibrium configurations of a Slinky via the minimization of its potential energy. Careful consideration of the contact between coils enables this procedure to accurately describe the shape and stability of the Slinky under different modes of deformation. In addition, we provide simple geometric and material relations that describe a scaling of the general behavior of flexible, helical springs.

The floppy nature of a tumbling Slinky (Poof-Slinky, Inc.) has captivated children and adults alike for over half a century. Highly flexible, the spring will walk down stairs, turn over in your hands, and – much to the chagrin of children everywhere - become easily entangled and permanently deformed. The Slinky can be used as an educational tool for demonstrating standing waves, and a structural inspiration due to its ability to extend many times beyond its initial length without imparting plastic strain on the material. Engineers have scaled the iconic spring up to the macroscale as a pedestrian bridge [1], and down to the nanoscale for use as conducting wires within flexible electronic devices [2, 3], while animators have simulated its movements in a major motion picture [4]. Yet, perhaps the most recognizable and remarkable features of a Slinky are simply its ability to splay its helical coils into an arch (Fig. 1), and to tumble over itself down a steep incline.

A 1947 patent by Richard T. James for "Toy and process of use" [5] describes what became known as the Slinky, "a helical spring toy adapted to walk and oscillate." The patent discusses the geometrical features, such as a rectangular cross section with a width-to-thickness ratio of 4:1, compressed height approximately equal to the diameter, almost no pretensioning but adjacent turns (coils) that touch each other in the absence of external forces, and the ability to remain in an arch shape on a horizontal surface. In the same year, Cunningham [6] performed some tests and analysis of a steel Slinky tumbling down steps and down an inclined plane. His steel Slinky had 78 turns, a length of 6.3 cm, and an outside diameter of 7.3 cm. He examined the spring stiffness, the effects of different step heights and of inclinations of the plane, the time length per tumble and the corresponding angular velocity, and the velocity of longitudinal waves. He stated that the time period for a step height between 5 and 10 cm is almost independent of the height and is about 0.5 s. Forty years later, he gave a further description of waves in a tumbling Slinky [7]. Longuet-Higgins [8] also studied a Slinky tumbling down stairs. His phosphor-bronze Slinky had 89 turns, a length of 7.6 cm, and an outside diameter of 6.4 cm. In his analysis, he imagined the Slinky as an elastic fluid, with one density at the end regions where coils touch and another for the rest. His tests produced an average time of about 0.8 s per step for a variety of step heights.

Heard and Newby [9] hung a Slinky-like spring vertically, held at its top, with and without a mass attached at the bottom. Using experiments and analysis, they investigated the length, as did French [10], Sawicki [11], and Gluck [12], and they studied longitudinal waves, as did Young [13], Bowen [14], and Gluck [12]. In the work by Bowen, the method of characteristics was utilized to obtain solutions of the wave equation (see also [15]), and an effective mass of the Slinky was discussed, which was related to the weight applied to an associated massless spring and yielding the same fundamental vibration period. Mak [16] defined an effective mass with regard to the static elongation of the vertically suspended Slinky. Blake and Smith [17] and Vandergrift *et al.* [18] suspended a Slinky horizontally by strings and investigated the behavior of transverse vibrations and waves. Longitudinal and transverse waves in a horizontal Slinky were examined by Gluck [12]. Crawford [19] discussed "whistler" sounds produced by longitudinal and transverse vibrations of a Slinky held at both ends. Musical sounds that could be obtained from a Slinky were described by Parker *et al.* [20], and Luke [21] considered a Slinky-like spring held at its ends in a U shape and the propagation of pulses along the spring. Wilson [22] investigated the Slinky in its arch configuration. In his analysis, each coil was modeled as a rectangular bar, and a rotational spring connected each pair of adjacent bars. Some bars at the bottom of each end (leg) of the arch were in full horizontal contact with each other due to the pretensioning of the spring. The angular positions of the bars were computed for springs with 87 and 119 coils, and were compared with experimental results. Wilson also lowered one end quasi-statically until the Slinky tumbled over that end. The discrete model in the present paper will be an extension of Wilson's model and will include rotational, axial, and shear springs connecting