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Static and dynamic instability analysis of composite cylindrical shell panels subjected to partial edge loading

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ABSTRACT

The postbuckling and dynamic instability behavior of simply supported composite cylindrical shell panels subjected to dynamic partial edge loadings and transverse patch loadings is studied in this paper considering von Kármán type of non-linearity. The stress distribution within the panel due to the applied partial edge loadings is evaluated by panel's membrane analysis. Subsequently using these stress distribution and via Hamilton's variational principle, the equations governing the instability behavior of shell panel are derived. Neglecting inertia terms, governing equations for the postbuckling analysis of panel are obtained. Galerkin's method is used in the solution procedure. It is observed from the postbuckling analysis that the cylindrical shell panel subjected to partial edge compression behaves as an imperfect shell panel as the partial edge compression in the x-direction induces tensile stress in the y-direction which makes the shell panel to deflect out-of-plane. It is also observed that by suitably adjusting the lamina number and lamina layup, the snap through behavior of shells can be altogether avoided. Dynamic instability regions of simply supported composite shell panels are traced by the method suggested by Bolotin. The linear and non-linear dynamic responses of the shell in stable and unstable regions are studied. This brings out various features of the instability problem such as, existence of beats and its dependence on forcing frequency and initial conditions, and effect of non-linearity on the response. It is found that for certain value of dynamic partial edge loading, the panel exhibits chaotic behavior.

1. Introduction

The load exerted on the aircraft wings or on the stiffened plate in the ship structures by the adjacent structural elements usually is non-uniform and the type of load distribution depends on the relative stiffness of the adjoining elements. The non-linear static and dynamic behaviors of plates and cylindrical shell panels under various loadings were discussed in many recently published papers [1–5]. The response of the structural elements under combined effects of vibration and buckling is obtained by various methods in Ref. [6]. In this book, the author discusses postbuckling, mode jumping, parametric excitation of structural elements. However, very few studies have been reported on the buckling analysis of plates subjected to non-uniform loads. The stability behavior of isotropic plates under localized edge load was studied [Baker and Pavolic [7], Brown [8]] using analytical methods. The exact solutions of vibration and buckling of rectangular plates under linearly varying loads were reported by Leissa and Kang [9,10]. Lopatin and Morozov [11] analytically studied the buckling of SSCF orthotropic plate subjected to linearly varying in-plane load. The authors first used the Kantorovich procedure to reduce the variational buckling equation to a one dimensional form and then applied the generalized Galerkin method. In this paper, an analytical formula for the calculation of buckling load under linearly varying in-plane load was also proposed. For the case of linearly varying load, the stress distribution within the plate coincides with the applied in-plane load distribution. The parametric instability of laminated composite plate and cylindrical shell panels subjected to non-uniform (parabolic) in-plane loading was studied by Ovesy and Fazilati [12]. The authors used the Koiter–Sanders shallow shell theory and finite strip method to investigate the dynamic instability behavior. The postbuckled vibration characteristics of thin plate such as secondary bifurcation and mode jumping phenomena are studied by Chen and Virgin [13,14] using the non-stationary finite element method. In the above papers, authors have shown how boundary conditions and loading types affect the post-secondary buckling form. The buckling of composite plates subjected to partial in-plane edge load was obtained by Sunderesan et al. [15] and Chakrabarty and Sheikh [16] using the finite element method. The buckling of plates subjected to non-uniform in-plane loading was studied

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by Bert and Devarakonda [17], Jana and Bhaskar [18]. The authors first solved the plane elasticity problem to determine the exact stress distribution within the plate and then solved for buckling load using the Galerkin method. Daripa and Singha [19] studied the postbuckling of composite plates under localized in-plane edge loads using the finite element method. The curved panels are very sensitive to initial geometric imperfections. In most cases, the first eigen mode with a pre-selected amplitude is used to model the initial geometric imperfections. Featherston [20] studied the influence of imperfection shape and amplitude on the buckling and postbuckling of curved panels subjected to combined shear and compression using the finite element method. The postbuckling behavior of anisotropic panels with initial geometric imperfections subjected to eccentric biaxial compression was studied by Romeo and Frulla [21]. Recently, the postbuckling analysis of laminated composite cylindrical shell panels with initial geometric imperfections and subjected to non-uniform edge loading and uniform lateral load was studied by Panda and Ramachandra [22] using Galerkin's method.

In this paper, the static and dynamic stability behavior of laminated composite cylindrical shell panels subjected to partial edge loading and transverse patch loading is presented. To the best of authors' knowledge, this is the first attempt to obtain postbuckling and dynamic instability analysis of panels subjected to partial edge loading and lateral patch loadings using the semi-analytical method. Initially, the panel's membrane problem is solved to evaluate the stress distributions within the panel in the prebuckling range due to these partial edge loadings. Subsequently, using the above stress distributions and via Hamilton's variational principle, the governing partial differential equations describing the stability behavior of cylindrical shell panels are derived. For tracing the panel's postbuckling path, the inertia terms in the above equations are neglected. Employing Galerkin's method these partial differential equations are reduced into a set of non-linear algebraic equations. Adopting the Newton–Raphson method and Riks approach, these algebraic equations are solved. Numerical results are presented for various loading combinations and imperfection magnitudes.

The dynamic instability regions of a simply supported composite cylindrical shell panel subjected to dynamic partial in-plane loads are studied neglecting non-linearity. Employing Galerkin's approximation, the governing partial differential equations are converted into a set of ordinary differential equations (Mathieu type of equations), which describe the panel's dynamic instability. Adopting the Fourier series method, the instability regions are determined from boundaries of instability, which represent the periodic solution to the Mathieu equations. The characteristics of instability regions are studied by the linear and non-linear responses of the shell in stable and unstable regions, which brings out various features of the instability problem such as, existence of beats and its dependence on forcing frequency and initial conditions, and effect of non-linearity on the response. Also, the existence of chaotic response is explored for certain values of dynamic partial edge loading.

2. Theoretical development

2.1. Kinematic equations

Consider a cylindrical shell panel of radius R and thickness h with rectangular plane form of length a and width b and made up of finite number of orthotropic layers of equal thickness. The orthogonal co-ordinate system (x, y and z) is taken at the mid-surface of the shell panel as shown in Fig. 1. In the present investigation the governing equations of cylindrical shell panel

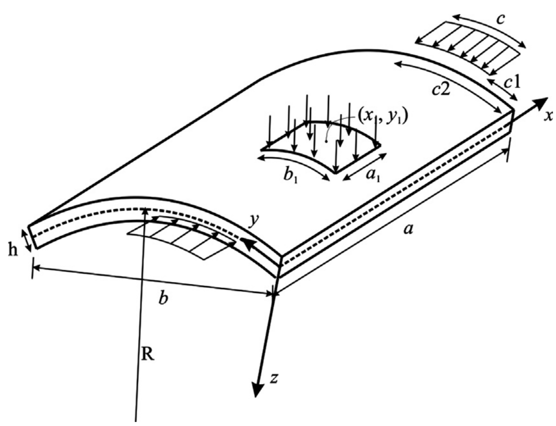

Fig. 1. Cylindrical shell panel with the co-ordinate system (x, y, z).

are derived based on Donnell's shell theory considering first order shear deformation theory. In the Donnell's theory the kinematic relations can be written as,

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon^0_{xx} \\ \varepsilon^0_{yy} \\ \gamma^0_{xy} \end{Bmatrix} + z \begin{Bmatrix} \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix} \tag{1}$$

ε_{xx}, ε_{yy} and γ_{xy} are the normal and shear strains in the x–y plane, and κ_{xx}, κ_{yy} and κ_{xy} are the curvatures. The superscript '0' used to represent the strain components (ε^0_{xx}, ε^0_{yy} and γ^0_{xy}) at the mid-surface of the shell panel. The mid-surface strains and curvatures are represented as,

$$\{\varepsilon^0_i\} = \begin{Bmatrix} \varepsilon^0_{xx} \\ \varepsilon^0_{yy} \\ \gamma^0_{xy} \end{Bmatrix} = \begin{Bmatrix} u^0_{,x} + \frac{1}{2} w^{02}_{,x} \\ v^0_{,y} + \frac{w^0}{R} + \frac{1}{2} w^{02}_{,y} \\ u^0_{,y} + v^0_{,x} + w^0_{,x} w^0_{,y} \end{Bmatrix};$$

$$\{\kappa_i\} = \begin{Bmatrix} \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix} = \begin{Bmatrix} \varphi_{x,x} \\ \varphi_{y,y} \\ \varphi_{x,y} + \varphi_{y,x} \end{Bmatrix} \quad (i = 1, 2, 6)$$

$$\{\varepsilon^0_i\} = \begin{Bmatrix} \gamma^0_{xz} \\ \gamma^0_{yz} \end{Bmatrix} = \begin{Bmatrix} \varphi_x + w^0_{,x} \\ \varphi_y - \frac{v}{R} + w^0_{,y} \end{Bmatrix} \quad (i = 4, 5) \tag{2}$$

u^0, v^0 and w^0 respectively represent the displacement components at the mid-surface of the shell along the x, y and z directions. φ_x, φ_y denote the rotations of cross-sections about y and x axes respectively. In the present analysis the geometric imperfections (w^*) are only considered in radial direction (w) of the cylindrical shell panel. The shell is assumed to be stress free in the presence of initial geometric imperfection. The strains due to the initial geometric imperfections may be expressed as,

$$\varepsilon^*_{xx} = \frac{1}{2}(w^*_{,x})^2; \quad \varepsilon^*_{yy} = \frac{1}{2}(w^*_{,y})^2; \quad \gamma^*_{xy} = w^*_{,x} w^*_{,y};$$
$$\gamma^*_{xz} = w^*_{,x}; \quad \gamma^*_{yz} = w^*_{,y} \tag{3}$$

The resultant strain components in the middle surface of the cylindrical shell become,

$$\varepsilon_{xx} = \varepsilon^0_{xx} + w^0_{,x} w^*_{,x} + z\varphi_{x,x}$$
$$\varepsilon_{yy} = \varepsilon^0_{yy} + w^0_{,y} w^*_{,y} + z\varphi_{y,y}$$
$$\gamma_{xy} = \gamma^0_{xy} + [w^0_{,x} w^*_{,y} + w^*_{,x} w^0_{,y}] + z[\varphi_{x,y} + \varphi_{y,x}] \tag{4}$$

The stresses, $\{\sigma\}^T_k = \{\sigma_{xx}, \sigma_{yy}, \tau_{xz}, \tau_{yz}, \tau_{xy}\}_k$, are related to strains, $\{\varepsilon\}^T_k = \{\varepsilon_{xx}, \varepsilon_{yy}, \gamma_{xz}, \gamma_{yz}, \gamma_{xy}\}_k$ for the orthotropic lamina by the relationship

$$\{\sigma\}_k = [\overline{Q}]_k \{\varepsilon\}_k \tag{5}$$