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Buckling phenomena in double curved cold-bent glass

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A B S T R A C T

Double curved anticlastic glazed surfaces are widely used for free-form façades and roofs of modern buildings. An effective technique consists in cold bending rectangular glass plies by twisting them with forces applied at the corners. The linear Kirchhoff–Love theory predicts that the deformed shape is a hyperbolic paraboloid, which preserves the straightness of the edges. However, experiments have provided evidence that a particular form of instability occurs above a certain limit of the distortion: one of the principal curvatures becomes dominant with respect to the other, the plate bulges into an asymmetric configuration and the edges are not any more straight. Here, a simple model is presented that, using a modified version of Mansfield's inextensional theory for thin plates, is able to interpret this phenomenon. Results are in good agreement with numerical experiments using large deflection theory. Moreover, the possibility of increasing the limit of the stable configuration by stiffening the edges is investigated.

1. Introduction

Curved glass is a powerful tool of aesthetic design for its transparency and capacity of diffusing the light and delimit a place without the barriers of a flat wall. Because of this, its use characterizes the most recent architectural trend towards twisted and free-form shaped surfaces. The number of constructions with transparent curved envelops is constantly increasing, including tall buildings, airports, exhibition areas, museums, concert halls and shopping arcades. In most applications, for safety reasons, glass has to be laminated with polymeric interlayers with a process at high pressure at temperature in an autoclave that produces a chemical bond between glass and polymer. Curving of laminated glass represents an additional difficulty, but its use is often imposed by building codes because, in case of rupture, the interlayer retains the glass shards avoiding major damage to property and human life. The difficulty increases even more so in the case of multiple-glazed envelopes, composed of several plies separated by spacers and glued at the borders, which are necessary to limit thermal loss and increase the energetic efficiency of the building. There is an increasing need for the massive production of curved panels for high-performance glazing, which represents the current challenge for both the industry and the designer.

Curved glass is traditionally produced through hot-forming processes. In *sag bending*, flat plates are placed on a negative curved mould and then heated up to a temperature at which glass softens and becomes plastic; the action of gravity produces the uniform contact with the support and, when glass is cooled down, it retains the shape of the mould. Another technique is *press bending*, where glass is formed by using a stainless steel mold face and head that presses the panel into the shape of the mold. In any case, the size of curved glass is limited by the size of the ovens, while every shape requires a dedicated mould, making this process attractive only for large quantities of *identical* panels. Moreover, glass cannot be hot-formed after lamination, because the polymeric interlayer cannot withstand the operating temperatures. Therefore, glass plies have to be hot-curved first and successively bonded together through the interlayer, but this process is not simple, requires very strict tolerances, and the greater encumbrance of the curved laminates with respect to the flat ones requires more space in the autoclave and, consequently, limits the production rate. In the case of multiple-glazed units, the bonding between the curved panels through the spaces requires even stricter tolerances and necessitates a special apparatus.

Cold bending of glass is a relative recent fabrication process. Flat glass panes are brought to the desired geometry by means of external contact forces, which hold the curved glass unit in the desired form. Consequently, glass can be curved directly at the construction site, holding it in place by clamping strips. This technique is increasingly developing because it has at least two major advantages. First, it does not need any negative template and, secondly, the degree of curvature can be easily modified through a slight variation of the constraining action. This allows for the construction, at relatively low cost, of curved free-form surfaces where all the glass panels may have slightly different geometries [1–3]. Moreover, the tolerances required to fit with the

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underlying load-bearing structure are less than in the case of pre-curved glazing. There is no additional difficulty in cold bending laminated glass, and also the production of multiple-glazed units is simplified because the unit can be bent after all the elements have been assembled while flat.

It should also be mentioned that another technique consists in bending glass layers in factory and laminate them with interlayers in the strained configurations, so that, after cooling, it is the bond of the interlayer that keeps the resulting sandwich in the desired shape. This procedure, usually referred to as *cold lamination bending*, has been discussed at length in [4].

Cold bending of glass greatly simplifies the production, but requires a careful calculation of the deformation and stress state in the panel in the service of a larger design objective. The simplest forms that can be obtained through cold bending are single-curvature developable surfaces. Recent advances in theoretical algorithms allow for the discretization of surfaces using only single-curvature pieces to reach double-curvature glazing of any form [5,6], but this solution has its limits, because the discretization may lead to modifications of the initial shape in order to be able to panelize. Synclastic double-curvature panels can be formed, but the curvature must be very low. On the other hand, anticlastic surfaces can be readily achieved by twisting initially flat panels. Indeed, this double-curvature form is the most feasible by cold-bending. Anticlastic-bent elementary cells may be considered a key element because they can be used to tessellate non-flat glazed surfaces [7], maintaining the desired smoothing.

In order to achieve large curvatures while maintaining the stress under the limit strength of the material, very thin glass panels need to be used in cold bending, with the consequent risk of loss of elastic stability (buckling). The purpose of this paper is to study possible instability phenomena in one of the most-used configurations for anticlastically bent glazed cells: the hyperbolic-paraboloid shape. Examples of recently built smooth curved façades that make use of this concept [8] are shown in Fig. 1. The advantage of this shape consists in the wide range of different surfaces that can be obtained through the repetition, with minor changes, of such an elementary unit, as well as in the ready applicability to this case of cold-forming techniques by simply twisting the plate.

The hyperbolic paraboloid is a quadric surface shaped like a saddle; it is doubly ruled so that at every point there is a couple of straight lines that lies on the surface. This property is very important because the classical linear Kirchhoff–Love theory predicts that if a flat rectangular elastic plate is twisted by out-of-plane forces applied at the corners, the deformed shape is actually a hyperbolic paraboloid [9], so that the plate edges, as well as the fibers initially parallel to the edges, remain straight. This greatly facilitates the cold-forming process, the forcing, the connection, the assembly and the installation (see, among others, [10,11]). However, an intriguing form of instability has been observed during the production when the strains become sufficiently large. In practice, above a certain value of the twisting angle, the deformed configuration tends to lose its symmetry: one of the diagonals straightens, while the curvature increases in the direction of the second diagonal, with the edges that bend considerably.

Although the phenomenon has been experimentally observed and recorded by many authors (see, e.g., [12,7,13,8]), to our knowledge it has never been studied on a theoretical basis. Here, the problem is analyzed in detail in the paradigmatic case of a square plate twisted by forces applied at the corners. Apart from numerical experiments, an analytical model is presented that improves Mansfield's inextensional theory for thin plates [14]. The model is useful to explain the mechanism of instability and, despite its simplicity, gives results in good agreement with numerical experiments. The onset of instability is clearly identified by the loss of symmetry in the deformation and by the curvature of the plate edges. The possibility of increasing the limit of the stable configuration by stiffening the borders with straight, bending-rigid, profiles is explored.

2. Anticlastic cold bent glass. Numerical experiments

In cold bent glass, the hyperbolic-paraboloid shape [8,15] is obtained by twisting the plate, forcing its corners in the desired

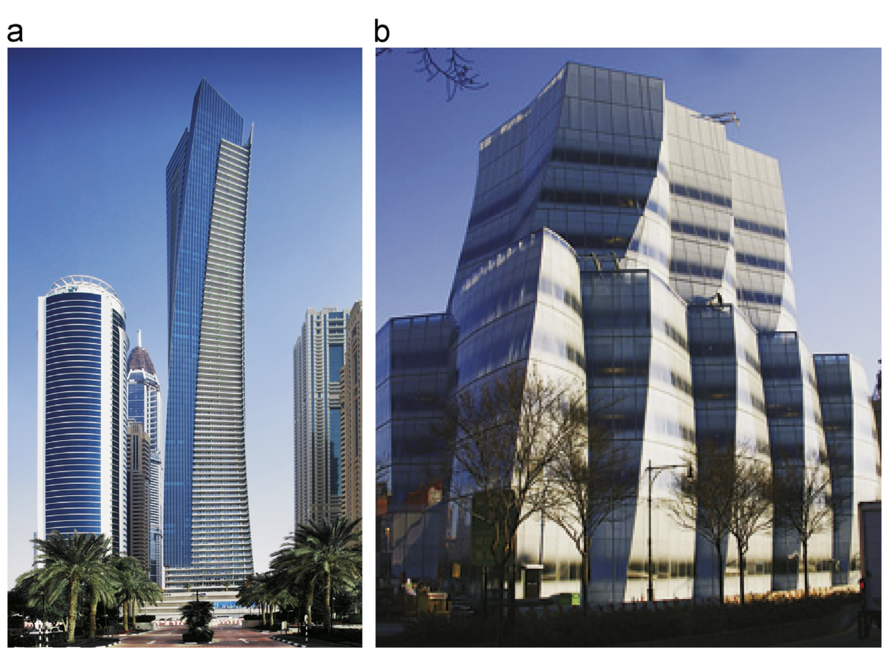

Fig. 1. Example of application of cold-bent glass panels in a hyperbolic paraboloid shape. (a) Ocean Heights Tower in Dubai, by Andrew Bromberg (2010); (b) IAC headquarters, New York City, by Frank Gehry (2007).