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Theoretical and numerical study of targeted energy transfer inside an acoustic cavity by a non-linear membrane absorber

Jianwang Shao [a,b], Bruno Cochelin [b,*]

[a] Tongji University, 201804, Shanghai, China
[b] LMA, UPR CNRS 7051, Centrale Marseille, F-13402 Marseille Cedex 20, France

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ABSTRACT

This paper investigates the Targeted Energy Transfer (TET) between an acoustic medium inside a parallelepiped cavity and a thin viscoelastic membrane that is mounted on one wall of the cavity and that is working as a Non-linear Energy Sink (NES). Applying a harmonic source excitation inside the cavity, we look for reducing the resonance peaks of the acoustic cavity by mean of the so-called "Strongly Modulated Response" (SMR). Using a single term harmonic balance approach, we determine analytical simplified expressions for the non-linear normal modes (NNMs) and the periodic forced responses of this coupled system. We define the "desired working zone" for the NES as the forcing level interval between the first destabilization of the resonance peak (TET beginning) and the appearance of an additional branch of periodic regimes. An analytical formula is derived for the beginning of TET while the appearance of the undesired periodic regimes is obtained numerically. These results prove to be useful for the practical design of NES to be mounted on an acoustic cavity.

1. Introduction

A new passive technique for reducing noise and vibration is to use purely non-linear absorbers working on the principle of "Targeted Energy Transfer" or "energy pumping". The concept of TET was first proposed by Vakakis and Gendelman [1,2], in 2001. We often speak of the Non-linear Energy Sink for a purely non-linear absorber.

The dynamics of a non-linear absorber were described in detail in [2,3] in terms of resonance capture and non-linear normal modes. Compared to classical passive systems, the non-linear absorber has the advantage of working in a wide frequency band. The first experimental demonstration of the TET phenomenon was published in 2005 [4]. Since then, several theoretical and experimental studies have been made in view of applications in the field of mechanical vibrations [5–8]. Recently the NES concept found successful implementations not only in vibration attenuation of the unwanted mechanical vibrations, but also in the acoustic field where the aforementioned methodology is used for a passive sound control in low frequency domain. Cochelin et al. [9] and Bellet et al. [10] have conducted a comprehensive study on the use of a thin viscoelastic membrane for the design of a NES in acoustics. They experimentally investigated the TET between the membrane and the acoustic medium for both free and forced oscillations. Mariani et al. [11] conducted a complementary study to show that a loudspeaker working outside its linear regime can also be an efficient NES. In these studies [9–11], the first acoustic mode of a long tube stands for the linear system to be controlled, and a weak coupling is set between the tube and the NES by meaning of a large coupling box. These configurations directly reproduced the so-called "grounded configuration" that was used in the pioneer papers by Gendelman and Vakakis for analyzing the pumping [12].

In view of applications in the acoustic field, one has to consider more general geometry for the acoustic medium, and for instance, 3D acoustic cavities should be considered. An important question is then "If a NES (membrane or loudspeaker) is mounted directly on the wall of a cavity, can we still observe TET between the NES and the acoustic modes of the cavity?" The answer is definitely positive [13] provided that the placement of the NES on the wall involves a weak coupling between the NES and the considered acoustic mode.

In this paper, we analyze the TET phenomenon between one given acoustic mode of a parallelepiped cavity and a membrane NES mounted on the wall of the cavity. We concentrate on the case where a harmonic source excitation is applied inside the cavity and we look for the reduction of the resonance peak by mean of the so-called "Strongly Modulated Response". This particular type of response is characterized by a repetitive occurrence of the TET which permits to limit the response amplitude of the acoustic mode near its resonance frequency.

It is now well established that the NES has an interesting self-tuning property: it is able to reduce several resonance peaks in a given

* Corresponding author.
E-mail addresses: shjw.lmj@gmail.com (J. Shao), bruno.cochelin@centrale-marseille.fr (B. Cochelin).

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frequency band. On the contrary, the level of forcing for which the NES can work well is limited and in the view of designing acoustic NES for the cavity, it is very important to be able to predict this range of forcing level analytically. Finding the threshold value of the forcing level beyond which SMR begins has been recently addressed in [14,15] by means of theoretical analysis with multiple time scales and complexification method. The results are however hardly transposable for our specific grounded system used in [10]. Here we adopt a simplified approach by looking for the first destabilization of the resonance peak under harmonic forcing to determine the level of forcing beyond which SMR begins. By using a single term harmonic balance approach for finding the NNMs and the periodic forced responses, we manage to derive an analytical formula for the level of forcing beyond which SMR begins. Moreover, when the level of forcing becomes higher, an additional branch of periodic regimes appears with higher amplitudes. This is one of the major drawbacks of using NES as a vibration and sound absorber reported in several recent works. Here we also address the level of forcing for the appearance of the undesired periodic regimes, but by a numerical approach. The two levels of forcing will be called hereafter "the threshold for the beginning of TET" and "the threshold for the appearance of undesired periodic regimes", and the zone of forcing between the two thresholds will be called "the desired working zone for the NES".

The structure of the paper is as follows. In Section 2, the acoustic cavity and the membrane absorber are modeled as a system with two degrees of freedom (DOFs) submitted to an harmonic forcing. The NNMs, the periodic forced responses and their stability are analyzed. Numerical simulations are also performed to verify the analytical predictions. In Section 3, we define the desired working zone for the NES, establish an analytical formula of the threshold for the beginning of TET and obtain the threshold for the appearance of undesired periodic regimes by a numerical approach. Section 4 contains some concluding remarks.

2. Targeted energy transfer under harmonic forcing

2.1. Description of the system

The system considered in this paper is composed of a primary linear system coupled by a non-linear system NES as shown in Fig. 1. The primary linear system is an acoustic medium inside a parallelepiped cavity with dimensions L_x, L_y and L_z. We assume that all the walls are rigid. The NES is a thin viscoelastic membrane that is mounted on one wall of the cavity. The eigenfrequencies of the acoustic cavity and the acoustic pressure of a mode marked by the integers l, m and n are described by the following form:

$$f_{lmn} = \frac{c_0}{2}\sqrt{\left(\frac{l}{L_x}\right)^2 + \left(\frac{m}{L_y}\right)^2 + \left(\frac{n}{L_z}\right)^2},$$

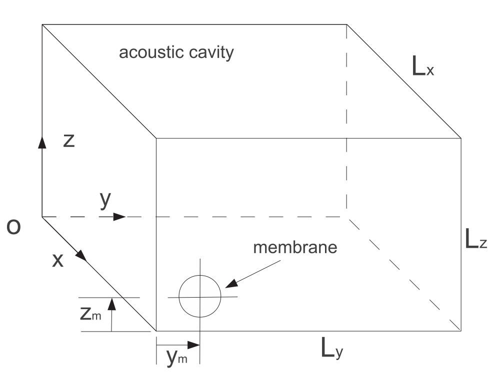

Fig. 1. Schema of the acoustic cavity with a membrane.

$$p_r(x,y,z,t) = P_{lmn}(x,y,z)p(t) = \cos\left(\frac{l\pi x}{L_x}\right)\cos\left(\frac{m\pi y}{L_y}\right)\cos\left(\frac{n\pi z}{L_z}\right)p(t), \quad (1)$$

where P_{lmn} is the mode shape and $p(t)$ is the acoustic pressure amplitude. The position of the membrane center is defined as (x_m, y_m, z_m), ($x_m = L_x$ in Fig. 1).

We assume that the first few modes of the cavity are separated in frequency, and we focus on the interaction between one mode of the acoustic cavity and the membrane. To analyze the TET phenomenon, we shall consider a simplified model with two DOFs system: one for the acoustic cavity and another one for the NES. This simplified model is assumed to be sufficient to analyze the influence of the membrane parameters and the influence of the membrane position (x_m, y_m, z_m) on the TET phenomenon.

For the equation of the membrane, we take the one DOF model of Bellet [10] but without the term of linear stiffness for simplicity (without the pre-stress). In [10], the membrane was mounted on a coupling box where the acoustic pressure was uniform. This is no more the case here, but if the size of the membrane is small compared to the size of the cavity and if we consider the first low frequency mode, we can assume that the acoustic pressure in contact with the membrane is uniform and equal to the value at the membrane center. It should be noticed that, again as in [10], the effect of the external pressure, i.e., outside the cavity, is not taken into account in the present study. The equation of the membrane is then of the following form:

$$m_m\ddot{q} + k_1\eta\dot{q} + k_3(q^3 + 2\eta q^2\dot{q}) = \frac{s_m}{2}P_{lmn}(x_m, y_m, z_m)p(t), \quad (2)$$

with

$$s_m = \pi R^2, \quad m_m = \frac{\rho_m h s_m}{3}, \quad k_1 = \frac{1.015^4\pi^5}{36}\frac{Eh^3}{(1-\nu^2)R^2},$$

$$k_3 = \frac{8\pi Eh}{3(1-\nu^2)R^2},$$

$$P_{lmn}(x_m, y_m, z_m) = \cos\left(\frac{l\pi x_m}{L_x}\right)\cos\left(\frac{m\pi y_m}{L_y}\right)\cos\left(\frac{n\pi z_m}{L_z}\right). \quad (3)$$

$q(t)$ is the transversal displacement of the membrane center (direction Ox in Fig. 1) and $p(t)$ is the acoustic pressure amplitude. The coefficients k_1 and k_3 stand for the linear and non-linear stiffness, respectively. R and h are the radius and the thickness of the membrane, respectively. E, ν, η and ρ_m are Young's modulus, Poisson's ratio, the viscous parameter and the density of the membrane, respectively.

The acoustic pressure inside the cavity is governed by the following equations:

$$\frac{\partial^2 p_r}{\partial t^2} - c_0^2\Delta p_r = 0 \quad \text{in } \Omega,$$
$$\frac{\partial p_r}{\partial n} = 0 \quad \text{on } \partial\Omega_1,$$
$$\rho_a\frac{\partial^2 w}{\partial t^2} = -\frac{\partial p_r}{\partial n} \quad \text{on } \partial\Omega_m \quad (\partial\Omega = \partial\Omega_1 + \partial\Omega_m), \quad (4)$$

where Ω is the internal volume of the acoustic cavity, $\partial\Omega$ is the surface of the acoustic cavity, $\partial\Omega_1$ is the surface of the acoustic cavity without the surface of the membrane, $\partial\Omega_m$ is the surface of the membrane. Finally, ρ_a is the density of the air and c_0 is the sound wave velocity.

From this equation, we perform a Rayleigh–Ritz reduction to one DOF system by using the mode shape P_{lmn} as a single shape function for p_r and for the test function $\delta p_r(x,y,z) = P_{lmn}(x,y,z)\delta p$:

$$\int_\Omega\left(\frac{\partial^2 p_r}{\partial t^2}\delta p_r - c_0^2\Delta p_r\delta p_r\right)d\Omega = 0,$$
$$\Rightarrow m_a\ddot{p} + k_a p = -\frac{\rho_a^2 c_0^2 s_m}{2}P_{lmn}(x_m, y_m, z_m)\ddot{q}, \quad (5)$$