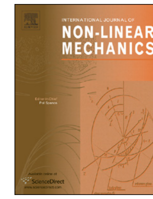
International Journal of Non-Linear Mechanics ■ (■■■■) ■■■–■■■

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm



Non-linear bending analysis of shear deformable functionally graded rotating disk

M. Kadkhodayan [a,*], M.E. Golmakani [b]

[a] Department of Mechanical Engineering, Ferdowsi University of Mashhad, Mashhad, Iran
[b] Department of Mechanical Engineering, Mashhad Branch, Islamic Azad University, Mashhad, Iran

ARTICLE INFO

Article history:
Received 4 December 2010
Received in revised form
22 June 2013
Accepted 23 August 2013

Keywords:
Functionally graded rotating disk
Non-linear bending
DR method

ABSTRACT

This study investigates the non-linear analysis of functionally graded solid and hollow rotating axisymmetric disks with uniform and variable thicknesses subjected to bending load. The material properties of the constituent components of the FG disk are assumed to be represented by the Mori–Tanaka distribution along the radial direction. The non-linear formulations are based on first-order shear deformation theory (FSDT) and the large deflection von-Karman equations. The dynamic relaxation (DR) method combined with the finite difference discretization technique is employed to solve the equilibrium equations. The effects of the grading index, angular velocity, geometry, thickness-to-radius ratio and thickness profile of the disk are studied in detail.

1. Introduction

Because of the importance of rotating disks in engineering applications such as in steam and gas turbines, flywheel systems, brake disks and clutches, researches on the behavior of these basic structures have been never stopped. In the all mentioned samples, body forces and bending loads are applied to a hollow or solid rotating disk. For example, the pressure difference across the rotors, as well as rotational stresses, causes bending in gas turbine rotors. Moreover, the force responsible for maintaining contact between the rotating disks leads to the bending in clutches [1]. These samples highlight the role of bending load in the design and analysis of rotating components. In all these applications, the performance of the components considering efficiency and service life depends on the material properties, geometry, boundary conditions and applied loads. Generally, these components can be manufactured using any metal. However, for some specific applications, they are fabricated using special materials. In the latter half of 20th century, composite materials have gained wide usage in engineering applications because of their benefits, including high strength. These structures have some disadvantages because of the different physical and mechanical properties of the two different layers and the discontinuity at the interface [2]. A new group of materials called functionally graded materials (FGMs) was introduced in 1984 by a group of material scientists in Japan to overcome these disadvantages [3]. FGMs made of ceramic and metal are capable of both suffering from high-temperature environment because of better thermal resistance of the ceramic phase [4] and being toughened by the metallic composition. Because of the advantages of FGMs over conventional composites and monolithic materials, these materials have been extensively studied for potential applications as structural elements, such as FGM beams, plates, shells, and cylinders [5]. FGMs belong to a new class of materials which are microscopically heterogeneous and their material properties vary continuously [6]. The material properties of FGMs show a continuous and smooth change from one surface to another by gradually changing the volume fraction of the constituent materials. Many studies on linear thermal bending of FGM plates are available in the literature. However, investigations in non-linear analysis of FGM plates under thermal or mechanical loadings are limited in number [7]. Because of the increasing use of FGMs, it is important to understand the non-linear behavior of functionally graded rotating disks under transverse loads. Many studies on the linear and thermoelastic behavior of rotating FG disks without consideration of transverse loads have been performed based on different plate theories. Durodola and Attia [8] conducted a finite element analysis for FG rotating disks using commercial software. Afsar and Go [5] carried out the finite element analysis of different thermoelastic fields in a thin circular FG rotating disk. Several works presented semi-analytical and thermoelastic solutions for hollow and solid rotating axisymmetric disks made of functionally graded materials under plane stress conditions [9–12]. In their works, a set of linear algebraic equations were obtained by dividing the radial domain into some virtual sub-domains. Ghafoori [13] expressed the shortcomings of study done by Hosseini Kordkheili and Naghdabadi [9]. They obtained the Navier thermoelastic equation and applied several incorrect coefficients, which resulted in considerable errors in their numerical results. In all the studies mentioned, the effect of the transverse load on the linear behavior of rotating disks was not considered. Recently, a few studies were conducted on the small deflection of FG rotating disks with transverse loads by Bayat et al.

* Corresponding author. Tel.: +98 9153 111869.
E-mail addresses: kadkhoda@um.ac.ir (M. Kadkhodayan), m.e.golmakani@mshdiau.ac.ir (M.E. Golmakani).

0020-7462/$ - see front matter © 2013 Published by Elsevier Ltd.
http://dx.doi.org/10.1016/j.ijnonlinmec.2013.08.007

[14,15]. In their works, a semi-analytical solution was given for the bending analysis of FG rotating disks under different types of boundary conditions applied to solid and hollow disks with uniform and variable thicknesses based on first-order shear deformation theory (FSDT). Moreover, Bayat et al. [16] used new linear and non-linear equilibrium equations for FG axisymmetric rotating disk with bending and thermal loadings based on FSDT and von-Karman theories. In their study, the material properties of FGM disks were assumed to vary continuously through the thickness according to a power-law distribution of the volume fraction of the constituents. Sahraee [17] studied the effect of transverse shear strains through the thickness using classical and shear deformation plate theories. The simplest shear deformation plate theory, FSDT, can be classified, depending on whether or not the expansion of displacement components or stress components through the thickness of plate is assumed to be known a priori [17]. In order to consider the effect of shear deformations, analyzing a plate statically Reissner [18] developed a stress-based theory while Mindlin [19] by analyzing a plate dynamically developed a displacement-based theory which include the effects of transverse shear deformations and rotary inertia (for more details see [20–22]). Reddy et al. [20] considered axisymmetric bending and stretching of FG solid and annular circular plates using FSDT. Yang and Shen [21] studied non-linear bending analysis of shear deformable FGM rectangular plates subjected to transverse and in-plane loads under various boundary conditions and by the aid of a semi-analytical–numerical method. Using Reissner and von-Karman plate equations, Reddy and Huang [22] studied non-linear axisymmetric bending of variable thickness orthotropic annular plates. Variable thickness plates (or disks) have always been attractive for designers, and a lot of researches have been carried out on this subject [23]. The investigation of disks with varying thickness is important in applied engineering because such geometries can improve the material potential by decreasing the self-weight and enhancing the distribution of stress and displacement [1,12,24]. When the thickness along the radius of a disk varies, finding a closed-form solution becomes very difficult [24]. To the best of our knowledge, no study has used a non-linear analysis of FG disks with uniform and variable thickness subjected to a transverse load and an inertia force due to rotation. To be more specific, in this paper, a thin FG disk of variable thickness subjected to inertia and bending transverse loads under various boundary conditions is investigated. The material properties of the constituent components of the FG disk are assumed to be represented by the Mori–Tanaka distribution [25], a micromechanics-based method for determining effective material properties of the FGM, along the radial direction. Moreover, some studies are conducted by the simple rule of mixtures for comparison between these two schemes. The constitutive equations are obtained based on FSDT using the von-Karman theory for large deflections. The dynamic relaxation (DR) method along with the finite difference discretization technique is employed to solve the disk field equations. Numerical results are presented graphically to show how the grading index parameter, angular velocity, boundary conditions and thickness profile of the disk affect different parameters such as the resultant stresses and displacements.

2. Problem formulation

2.1. FGM material properties

There are some models to express the variation of material properties in FGMs in the literature. The functions are continuous, simple and able to represent realistic properties of actual FGMs. The most commonly used model is the power-law distribution of the volume fraction. However, the elastic properties of actual FGMs are very different than those estimated by simple rules of mixture. The prediction of the macroscopic stress–strain response of composite materials depends on the description of their complex microstructural behaviors represented by the interaction between the constituents. Therefore, using a micromechanics-based method for FGM's material properties such as Mori–Tanaka theory can represent the realistic prediction for the behavior of the FG disk. These schemes provide efficient and straight forward algorithms for the prediction of the elastic constants. These methods are based on two steps to predict the macroscopic response. In the first step, a local problem for a single inclusion is solved to obtain an approximate local field behavior and in the second step the global fields are obtained by averaging the local ones [26]. In the current study, the Mori–Tanaka scheme [25–27] was used for the modeling of elastic properties. Furthermore, to compare the results obtained by this theory and the simple rules of mixture, several studies were conducted using a power-law distribution. According to the simple rule of mixture, the material properties along the radius of the disk P can be expressed as

$$P(r) = (P_c - P_m)V_c + P_m\,, \tag{1}$$

where $P(r)$ denotes a generic material property and the P_m and P_c denote the properties of the metallic and ceramic constituents, respectively. The inner and outer surfaces of the FG disk, r_i and r_o, are assumed to be metal-rich and ceramic-rich, respectively. The volume fractions of the metal V_m and ceramic V_c corresponding to the power law are assumed to be

$$\begin{cases} V_c = \left(\frac{r-r_i}{r_o-r_i}\right)^n, \\ V_m = 1 - V_c, \end{cases} \tag{2}$$

where r is the radial coordinate ($r_i \le r \le r_o$) and n is a grading index that dictates the material variation profile. Accordingly, the assumed form for the modulus of elasticity E and mass density ρ is the following:

$$E(r) = (E_c - E_m)\left(\frac{r-r_i}{r_o-r_i}\right)^n + E_m, \tag{3}$$

$$\rho(r) = (\rho_c - \rho_m)\left(\frac{r-r_i}{r_o-r_i}\right)^n + \rho_m. \tag{4}$$

For many engineering materials, Poisson's ratio is within the range of 0.25–0.35 and varies slightly. Therefore, for simplicity, Poisson's ratio is assumed to be a constant in the following study. However, the variable Poisson's ratio along the radial direction does not give rise to any difficulty in determining the solution [28]. The effective bulk modulus, K, and the effective shear modulus, G, of the functionally gradient material based on the Mori–Tanaka homogenization method [27] are the following:

$$\begin{cases} \frac{K-K_c}{K_m-K_c} = \frac{V_m}{1+(1-V_m)\frac{K_m-K_c}{K_c+(4/3)G_c}}, \\ \frac{G-G_c}{G_m-G_c} = \frac{V_m}{1+(1-V_m)\frac{G_m-G_c}{G_c+f_c}}, \end{cases} \tag{5}$$

where

$$f_c = \frac{G_c(9K_c+8G_c)}{6(K_c+2G_c)}. \tag{6}$$

Based on this method, the effective values of Young's modulus E are computed using:

$$E = \frac{9KG}{3K+G}. \tag{7}$$

2.2. Governing equations

In this section, the fundamental equations of the large deflection analysis of pressure-loaded FG rotating disks are briefly outlined. Consider a thin, axisymmetric FG disk with variable thickness, inner radius r_i and outer radius r_o at a constant angular velocity ω about the

Please cite this article as: M. Kadkhodayan, M.E. Golmakani, Non-linear bending analysis of shear deformable functionally graded rotating disk, International Journal of Non-Linear Mechanics (2013), http://dx.doi.org/10.1016/j.ijnonlinmec.2013.08.007