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A non-linear higher-order thickness stretching and shear deformation theory for large-amplitude vibrations of laminated doubly curved shells

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ABSTRACT

A geometrically non-linear theory is developed for shells of generic shape allowing for third-order thickness stretching, higher-order shear deformation and rotary inertia by using eight parameters; geometric imperfections are also taken into account. The geometrically non-linear strain–displacement relationships are derived retaining full non-linear terms in the in-plane and transverse displacements and are presented in curvilinear coordinates, ready to be implemented in computer codes. Higher order terms in the transverse coordinate are retained in the derivation so that the theory is suitable also for thick laminated shells. The theory uses the three-dimensional constitutive equations and does not need the introduction of traction/compression free hypothesis at the shell inner and outer surfaces. The traction/compression free condition is introduced only to obtain a simplified model with six parameters instead of eight. The third-order thickness stretching theory is applied to cross-ply symmetrically laminated circular cylindrical shells complete around the circumference and simply supported at both ends. Geometrically non-linear forced vibrations are studied by using the present theory and results are compared to those obtained by using a refined higher-order shear deformation non-linear shell theory, which neglects thickness stretching, and to results obtained by using first-order and second-order thickness stretching theories. Results obtained by using the reduced third-order thickness stretching model with six parameters are also presented and compared.

1. Introduction

Significant thickness stretching is observed for shells made of soft materials, like rubbers or biological materials, during large deformations. Moreover, dynamic variation of the shell thickness can be observed in very high-frequency vibration of shells for specific natural modes.

It is known that classical shell theories, which are those neglecting shear deformation and rotary inertia, overestimate natural frequencies of moderately thick or laminated anisotropic shells and plates (e.g. [1]). In order to overcome this limitation, shear deformation theories have been introduced. These theories can be classified in first-order and higher-order shear deformation theories [2]; in the first category, a shear correction factor is required for the equilibrium since a uniform shear strain is assumed through the shell thickness. Higher-order shear deformation theories overcome this limitation since a realistic shear stress distribution through the shell thickness is considered; the condition of zero shear stresses at both top and bottom shell surfaces is also satisfied.

Several higher-order shear deformation shell theories have been developed over the years. Librescu [3] proposed a non-linear shell theory by expanding the shell displacements with cubic terms in the transverse coordinate. A linear higher-order shear deformation theory of shells has been introduced by Reddy [4] and Reddy and Liu [5]. Arciniega and Reddy [6] have improved the theory developed in [5]. A review of shell theories has been presented by Reddy and Arciniega [7].

Reddy [8] developed the non-linear higher-order shear deformation theory of plates, taking into account von Kármán type non-linear terms. Dennis and Palazotto [9] and Palazotto and Dennis [10] have extended Reddy shell theory [4] to non-linear deformation by introducing the von Kármán type non-linear terms. These theories have been also discussed in the books of Amabili [2] and Reddy [11]. In the existing higher-order shear deformation geometrically non-linear shell theories, the von Kármán type non-linear terms (i.e. those involving the normal displacement only) have been added to the linear equations, so that consistent derivation has not been performed. Moreover, the von Kármán type non-linear terms are known for being accurate for classical shell theories only for small

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displacements and moderately small rotations. Therefore, it is important to derive a consistent higher-order shear deformation theory that keeps all the non-linear terms in the normal and in-plane displacements. For this reason, a new non-linear higher-order shear deformation theory that retains in-plane non-linear terms has been recently derived by Amabili and Reddy [12] by using a consistent approach. This theory belongs to the class of the equivalent single layer (ESL) theories and has the novelty to retain non-linear terms with in-plane displacements, neglected in other formulations. It must be observed that another class of theories, the layer wise model (LWM) has also been developed, but not retaining geometrically non-linear terms, see e.g. Carrera [13].

Amabili [14] and Alijani and Amabili [15] have applied the theory developed in reference [12] to laminated closed and open circular cylindrical shells and have shown that it gives an important accuracy improvement for thick laminated deep shells, in particular for vibration modes with a low number of circumferential waves, with respect to commonly used non-linear higher-order shear deformation theories.

Other non-linear shell theories have been derived, as the Lagrangian non-linear theory of thin shells expressed in terms of displacement of the shell middle surface as only independent field variables [16].

Shear deformation theories, which are obtained under the hypothesis of zero transverse normal stress, are accurate unless the shell presents significant thickness stretching or high frequency vibrations involving thickness oscillation. Shells made of rubbers or biological materials can present very large deformations, still in the linear material regime or in hyperelastic regime, associated to large thickness stretching (e.g. arteries under internal pressure). In this case, it can be necessary to use a shell theory that takes into account for the thickness stretching. The effect of transverse normal stress in multi-layered plates and shells and their natural vibrations has been studied by Carrera [13,17] by using a model that considers continuity of interlaminar transverse and shear stresses and zig-zag form of the displacement distribution in the shell thickness.

Shell theories that take thickness stretching into account also overcome the inconsistency of the assumption of simultaneous zero transverse normal stress and strain introduced by classical and shear deformation theories. In fact, they use the full three-dimensional constitutive equations, without resorting to plane stress assumption.

An accurate linear shell theory that takes into account thickness stretching has been developed by Carrera et al. [18] and Ferreira et al. [19]. In particular, in reference [19] natural frequencies of laminated spherical and cylindrical panels are investigated by using this theory.

Büchter et al. [20] and Bischoff and Ramm [21,22] elaborated seven-parameter shell theories that introduces a linear and a quadratic term to describe the stretching of the shell thickness. These models have been developed in order to eliminate the inconsistency of assuming a zero (for classical and shear deformation theories) or constant (for first-order thickness variation theories) transverse normal stress through the thickness. The model has been derived for finite element formulation by using the enhanced assumed strain concept and attention has been placed in order to avoid thickness locking. Only static problems have been addressed.

Also Parisch [23] and Sansour [24] have been developed independently shell theories that introduce quadratic assumption of the shell displacement over the shell thickness.

A first-order shear deformation shell theory with seven parameters and exact non-linear deformations, under the framework of the Lagrangian description, has been derived by Arciniega and Reddy [25,26] for laminated and functionally graded shells. This theory also uses quadratic expansion of the shell displacement over the shell thickness. A similar theory has been used by Payette and Reddy [27] in conjunction with a high-order spectral/hp type quadrilateral finite element technology.

The theory developed by Amabili and Reddy [12] introduces the artificial constraint of no thickness deformation, reducing the problem to five variables (3 displacements and two rotations). However, they discussed that their new theory could be extended to take into account thickness stretching. This extension has been developed by Amabili [28] who introduced a first-order thickness stretching theory with higher-order shear deformation. The application of this theory to a graphite-epoxy laminated circular cylindrical shell shows no significant difference in case of linear and non-linear vibrations for the unloaded shell. However, non-linear vibrations of a shell initially pre-stressed by large uniform pressure show important difference if the thickness stretching is taken into account.

In the present study, a geometrically non-linear theory is developed for shells of generic shape allowing for third-order thickness stretching, higher-order shear deformation and rotary inertia by using eight parameters; geometric imperfections are also taken into account. This seems to be the first study using 8 parameters to describe the shell non-linear deformation. The geometrically non-linear strain–displacement relationships are derived retaining full non-linear terms in the in-plane and transverse displacements and are presented in curvilinear coordinates, ready to be implemented in computer codes. Higher order terms in the transverse coordinate are retained in the derivation so that the theory is suitable also for thick laminated shells. The theory uses the three-dimensional constitutive equations and does not need the introduction of traction/compression free hypothesis at the shell inner and outer surfaces. The traction/compression free condition is introduced only to obtain a simplified model with six parameters instead of eight. The third-order thickness stretching theory is applied to cross-ply symmetrically laminated circular cylindrical shells complete around the circumference and simply supported at both ends. Geometrically non-linear forced vibrations are studied by using the present theory and results are compared to those obtained by using the refined higher-order shear deformation non-linear shell theory developed by Amabili and Reddy [12], which neglects thickness stretching, and to results obtained by using first-order and second-order thickness stretching theories. Results obtained by using the reduced third-order thickness stretching model with six parameters are also presented and compared.

2. Non-linear theory with higher-order thickness stretching and shear deformation

Fig. 1 shows a laminated shell of arbitrary shape, made of a finite number of orthotropic layers, oriented arbitrarily with

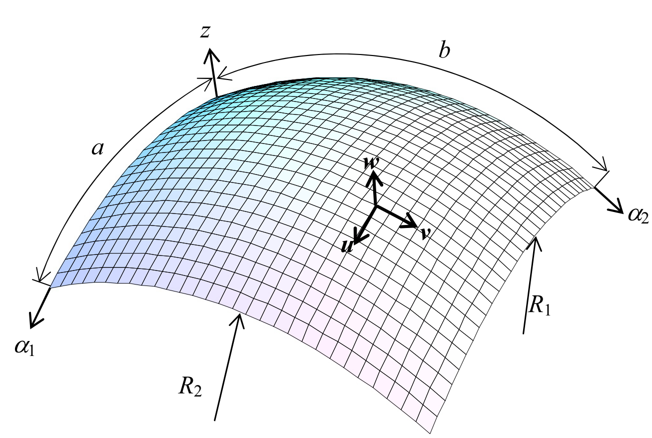

Fig. 1. Doubly curved shell and coordinate system.

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