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Post-buckling analysis of space frames using concept of hybrid arc-length methods

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ABSTRACT

Hybrid arc-length methods have been used for tracing the post-buckling equilibrium path of semirigid elastoplastic space frames. For example, the original implicit arc-length method uses the implicit Newton–Raphson method in both the predictor and the corrector steps, while the explicit arc-length method uses the explicit dynamic relaxation method in both the steps. The explicit and implicit arc-length methods have a clear disadvantage in that both require an excessive number of iterations, and the matrices are often singular. In this study, algorithms for implicit–explicit and explicit–implicit hybrid arc-length methods are developed for use in the predictor and corrector steps, to improve the accuracy and efficiency of the said method. The accuracy and applicability of the proposed methods are investigated by solving examples.

1. Introduction

The arc-length method (ALM) [1–5] is one of the most frequently used non-linear numerical techniques for stability analysis. It is very effective in finding the equilibrium path and the bifurcation point in buckling and post-buckling problems. The structural behavior and load parameters of a primary path can be obtained using the arc-length constraint given a predefined arc-length parameter. The ALM can be classified as either spherical [1] or cylindrical [2–5] based on the arc-length parameters used in the method. The spherical ALM is preferred for bifurcation problems involving complex equilibrium paths. However, this method requires asymmetric system–matrix operations.

Crisfield [2] and Ramm [3] proposed a cylindrical ALM and applied it to the generalized displacement method originally developed by Batoz and Dhatt [6]. The cylindrical ALM can be employed for determining load parameters using a simple quadratic function. Non-linear analysis can be performed effectively using the Newton–Raphson (NR) method because the system matrices are banded and symmetric, which confirms the quadratic convergence rate. However, the method has the drawback of unstable singularity near the critical point.

Despite its weakness, the implicit ALM with NR-type algorithms has been considered effective for solving bifurcation problems. In contrast to bifurcation problems, limit-point stability problems for snap-through and snap-back analyses do not necessitate the use of matrix-based eigenvalues or eigenvectors. Instead, eigenvalues/vectors are used as supplementary information.

To overcome the drawback of the NR method, Lee et al. [7] proposed a new explicit ALM by combining explicitly the cylindrical ALM and the dynamic relaxation method (DRM). The methodology does not require matrix operations and has been successfully applied to non-linear post-buckling problems. Consequently, the explicit ALM [7] without any matrix operations has advantages in terms of stability in solving limit-point stability problems. The algorithm can solve for displacements and load parameters simultaneously using the cylindrical arc-length constraint proposed by Crisfield [2] and Ramm [3] without any matrix-based instability or operations (such as eigenvalue problems). However, the number of iterations and the computing time required to obtain the converged solution might be moderately larger than those of the implicit NR process.

Conventionally, steel frames are analyzed and designed by idealizing existing boundary conditions to those of rigid- or hinge-connections under the assumption of linear elastic material properties. However, test results show that the connection behavior is similar to that of a semirigid or non-ideal joint. Yielding, cracking, and strain-hardening have significant effects on non-linear structural behavior, and accuracy could be affected by simplifying the conditions to only fixed ends or hinges. Furthermore, for buckling and

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post-buckling analyses, the element should include the formulation of geometric non-linearities such as rigid body deformation and finite joint rotation. Therefore, the semirigid nature of the boundaries, and the geometric and material non-linearities of the space frame should be considered appropriately. However, the elastoplastic post-buckling response of a semirigid jointed space frame has not been studied completely in the collapsed simulations.

In this paper, various algorithms of the explicit ALM and semirigid elastoplastic space element are described based on the previous research [7,8]. The original implicit ALM uses the implicit NR in both the predictor and the corrector steps, whereas the explicit ALM uses the explicit DRM in both the steps. The so-called hybrid ALMs described in this paper use the implicit or explicit algorithm in the predictor or corrector steps. As a result, the implicit–explicit and explicit–implicit hybrid ALM algorithms are developed in the predictor and corrector steps, respectively. The disadvantages of having an excessively large number of iterations in the explicit ALM and of matrix singularity in the implicit ALM can be resolved by using the mixed hybrid ALMs.

A space frame element that can simulate the material and geometrical non-linearities, and the semirigid boundaries [8] is further developed for accommodating the explicit algorithms of DRM and hybrid ALM. The beam-column equation is used in the element equilibrium equation with bowing effect [9], and rigid body motions and finite rotations of large displacements are considered in the Eulerian finite theory [10,11]. Plastic deformation due to material yielding is simply described by means of plastic hinges using perfect material plasticity. The elastic-connection spring formulation [12,13] is adopted for considering the semirigid connections of member ends. The overall equilibrium equation of a composed element can be derived using the static condensation technique. Numerical examples of elastoplastic post-buckling analysis with a semirigid connection property are performed using various explicit ALMs, and the numerical accuracy and efficiency of the methodology are discussed.

2. Semirigid elastoplastic beam-column element

This study uses a large deformational elastoplastic three-dimensional (3D) space frame element based on an Eulerian-formulated beam-column element [8]. The local member force–deformation relationships are based on the beam-column approach, and the changes in member chord lengths caused by axial strains and flexural bowing are considered [9]. A Eulerian formulation [10,11] that considers the effects of finite rotations of large joints is shown in Fig. 1. The joint element is added to that element using the static condensation process [12,13]. However, the element does not include the coupling effect of axial–torsional flexibilities and Wagner effects.

As shown in Fig. 1, if an old position $\mathbf{x}_{old}$ of a body is rotated to a new position $\mathbf{x}_{new}$, Euler's theorem of rigid body motions implies that any finite rotation can be described as a single rotation θ about some fixed axis described by a unit vector, $\mathbf{n}^* = \{n_1^*,\ n_2^*,\ n_3^*\}$. The new position vector $\mathbf{x}_{new}$ can be described using the old position vector $\mathbf{x}_{old}$, the rotation θ, and the fixed axis unit vector $\mathbf{n}^*$.

Based on Euler's finite rotation formula, the rotation matrix is necessary for describing both joint rotations and member rigid body rotations. This matrix can be derived from the so-called Rodriguez rotation vector, a rotation about a fixed axis represented by a unit vector, and a scalar angle of rotation.

Thus, a 3D rotation can be represented by a vector-like entity, but such entities cannot be added like vectors. Furthermore, it is assumed that these vector-like entities possess Taylor series expansions whose increments are the small rotation vectors obtained using linear structural analysis [23,26]. For separating the large rigid body deformations of a member from its relative deformations, which are

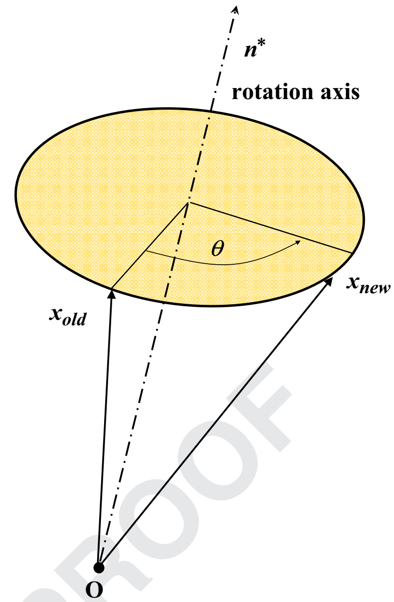

Fig. 1. Eulerian finite rotation about an axis.

assumed to be small, a Eulerian or local member coordinate system is used. In Fig. 2, the unit vector of the non-deformed member axis is $\mathbf{n}_i$, and the unit vector of the deformed member axis is $\mathbf{n}_i^{'}$. The rotation vector at joint 2 (θ_2) can be represented by the cross product of the member unit vector $\mathbf{n}_i$ and the deformed member unit vector $\mathbf{n}_2^*$ at joint 2 with the following properties:

$$\theta_2 \cong \mathbf{n}_i \times \mathbf{n}_2^* \tag{1}$$

$$|\theta_2| = \cos^{-1}(\mathbf{n}_i \mathbf{n}_2^*) \tag{2}$$

The end of member rotation vector for joint 1 (θ_1) can be expressed as follows:

$$\theta_1 = \theta_2 + \theta_{12} \tag{3}$$

where θ_{12} is the rotation of joint 1 with respect to joint 2. From this simple and exact formulation of Euler's finite rotation, we can calculate the exact member end rotations, θ_1 and θ_2, which separate the rigid body rotations.

The member force of the space frame element shown in Fig. 3 can be written using two-dimensional (2D) beam-column equations, which include the effect of member displacement on the bending moment to the end.

As shown in Fig. 3, $\theta_{1,j}$ and $\theta_{2,j}$ $(j = 2, 3)$ denote the relative member end rotations with respect to the X_j-axis coordinate system, as calculated using Eq. (3). φ_t and u denote linear axial twist and axial displacement, respectively. $M_{1,j}$, $M_{2,j}$, M_t, and Q denote the bending moments; twist moment; and member axial force corresponding to the relative member end rotations, axial twist, and axial displacement, respectively.

Consequently, the member force–deformation can be written as follows [9,10,11]:

$$\bar{\mathbf{s}}^T = \{Q : M_t : M_{1,2} : M_{1,3} : M_{2,2} : M_{2,3}\} \tag{4}$$

$$\bar{\mathbf{u}}^T = \{u : \varphi_t : \theta_{1,2} : \theta_{1,3} : \theta_{2,2} : \theta_{2,3}\} \tag{5}$$

The incremental form of the member equilibrium equation can be written as follows:

$$\Delta\bar{\mathbf{s}} = \mathbf{k}_b \Delta\bar{\mathbf{u}} \tag{6}$$

where $\mathbf{k}_b$ denotes the local stiffness matrix according to the incremental member displacement $\Delta\bar{\mathbf{u}}$ in the local coordinate

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