International Journal of Non-Linear Mechanics 58 (2014) 99–110

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Damage detection of an aeroelastic panel using limit cycle oscillation analysis

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ARTICLE INFO

Article history:
Received 9 July 2011
Received in revised form
8 September 2013
Accepted 19 September 2013
Available online 28 September 2013

Keywords:
Damage detection
Limit cycle amplitude
Feedback force
Modal method
Non-linear aeroelastic panel
Bifurcation boundary

ABSTRACT

Most of real systems are non-linear and they require damage detection. For such non-linear systems, non-linear damage detection methods are necessary for more accurate results. In this paper, a novel non-linear method is introduced using limit cycle oscillations that arise once the bifurcation (flutter) boundary is exceeded and shows greater sensitivity for damage detection versus linear damage detection techniques. Another advantage of this method is that it can be used for health monitoring of linear or non-linear systems. Here a non-linear aeroelastic panel is considered as a model to show the capabilities of the proposed damage detection technique. Also Proper Orthogonal Decomposition (POD) is used to find the number of independent damage locations in the panel. Rayleigh–Ritz method is used to discretizing spatial domain of the system. Damage is modeled as stiffness reduction in the certain region. By comparing Limit Cycle Oscillation of healthy and damaged panel damage level and its location could be obtained with good sensitivity.

1. Introduction

Detecting flaws, cracks and small amounts of damage in systems and structures are very important to prevent large and dangerous damage especially in aircraft and aerospace systems. Currently, there are several damage detection methods used for health monitoring of aircraft systems. For example laser ultrasonics and wavelet transform signal analysis is a technique used for hidden corrosion detection in aircraft aluminum structures [1]. Another method which is used for hidden corrosion detection of aircraft structures uses Lamb modes and ultrasonics or wave propagation method [2,3]. Stress waves and multivariate statistics were used by Mustapha to detect damage of an aircraft component [4]. X-ray backscatter is another method used for corrosion detection in aircraft [5]. Each method has its own advantages or drawbacks. Many of the above methods use high frequency content to track changes in the material. Unfortunately these methods can be used to detect only local damages and many of them cannot be used as a real time tool for damage detection.

In contrast to high frequency methods, there are vibration based damage detection techniques which use various vibratory features and low frequency data of systems to detect changes. These methods have different categories for example frequency or mode shape changes due to changes in stiffness, damping or mass of the system can be used to track damage [6–9].

Some methods based on frequency measurements can detect extent and damage location [10,11]. Measuring frequency is very simple and more accurate than measuring mode shape and modal damping, but it provides less information about damage and in many cases the detected extent or location of the damage is uncertain or even wrong. Thus methods based on mode shapes have been created [12–14]. Measuring flexibility and stiffness changes for structural damage localization is another method that uses vibrational data [15]. Identification of damage with measured frequency response functions is another method in this category [16].

An important point is that, all of these vibrational methods are well developed for linear systems and their sensitivity decreases for damage detection in non-linear cases, while many of the aeroelastic systems are non-linear and need non-linear damage detection techniques. So another category of vibrational damage detection based on vibrational data from non-linear systems has been developed.

For example, damage detection in non-linear systems using system augmentation [17], identification of damage in an aeroelastic system based on attractor changes [18], enhancing non-linear dynamics for accurate identification of stiffness loss in a thermo-shielding panel [19], using attractor dimension as a feature in structural health monitoring [20,21], and structural

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http://dx.doi.org/10.1016/j.ijnonlinmec.2013.09.008

Nomenclature

a, b = feedback force control parameters
$D=\frac{Eh^3}{12(1-\nu^2)}$ = panel stiffness
$\overline{D}$ = panel reduced stiffness in damaged portion
E = modulus of elasticity
$F(x,\tau)=\frac{l^4 P(X,t)}{Dh}$ = non-dimensional feedback force
$G=\frac{Eh^3}{2D}=6(1-\nu^2)$ = material parameter
h = panel thickness
l = panel length
M = Mach number
m, n = mode number
$q_n=\overline{q}_n/h$ = non-dimensional generalized coordinate
$R_x=\frac{Eh\eta_0 l^2}{D}$ = in-plane load parameter
$S_r=\overline{D}/D$ = stiffness reduction ratio
t = time
U_∞ = fluid velocity
$w=W/h$ = non-dimensional panel transverse deflection
$x=X/l$ = non-dimensional streamwise coordinate
$\lambda=\frac{\rho_\infty U_\infty{}^2 l^3}{MD}$ = non-dimensional dynamic pressure
$\mu=\frac{\rho_\infty l}{\rho_m h}$ = non-dimensional mass ratio
ν = Poisson's ratio
ρ_∞, ρ_m = air density, panel density
$\tau=t/\sqrt{\frac{\rho_m h l^4}{D}}$ = non-dimensional time
$w', \dot{w}$ = $\frac{\partial w}{\partial x}$, $\frac{\partial w}{\partial \tau}$

health monitoring through chaotic interrogation [22] are some of these methods that use vibrational data of a non-linear system for damage detection.

In 2005, Yin and Epureanu proposed a bifurcation boundary analysis method as a new non-linear damage detection tool and used it to track bifurcation boundary changes due to damages over a small region of an aeroelastic panel model [23]. They used a feedback force containing two parameters to plot the bifurcation boundary as a function of these parameters. In the previous work of Yin and Epureanu [23] and also the present authors, changes in the bifurcation boundary due to damage was the focus and a comprehensive study by the present authors showed that such changes are essentially insensitive to the damage [24]. In this article, the same feedback force is used get bifurcation boundary, but the focus in the present paper is on the limit cycle oscillations that arise once the flutter boundary is exceeded and their potential for damage detection. The results show higher sensitivity to small damage and it is possible to track the damage level and extent using this method. Finally Proper Orthogonal Decomposition (POD) is used to find the number of independent damage locations on the panel.

2. Damage detection technique using limit cycle oscillation (LCO) analysis

In order to use this method, first suppose that the linear or non-linear system is descretized (for example using a Finite Difference or Finite Element Method) so the equation of motion can be written as

$$M\ddot{\mathbf{x}}+C\dot{\mathbf{x}}+\mathbf{K}\mathbf{x}=\mathbf{F} \tag{1}$$

here **M**, **C** and **K** are mass, damping and stiffness matrices and **F** is force vector affecting the system. Also if the system is non-linear, one can include the non-linear terms in the **F** vector. Eq. (1) can be written in the state-space form and becomes

$$\dot{\mathbf{X}}=\mathbf{A}\mathbf{X}+\mathbf{B} \tag{2}$$

where **X** is the state vector and contains displacement and velocity vectors of the initial system or

$$\mathbf{X}=\begin{bmatrix}\mathbf{x}\\ \dot{\mathbf{x}}\end{bmatrix} \tag{3}$$

and the **A** and **B** matrices are as follows:

$$\mathbf{A}=\begin{bmatrix}\mathbf{0} & \mathbf{I}\\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C}\end{bmatrix}$$
$$\mathbf{B}=\begin{bmatrix}\mathbf{0}\\ \mathbf{M}^{-1}\mathbf{F}\end{bmatrix} \tag{4}$$

Also a feedback force is used to change the system behavior. This force is a function of the system states and also two parameters, for example, a and b. The feedback force used here is linear, so these two parameters will appear in the **A** matrix. Also it should be noted that if the initial system is linear, then the feedback force can be chosen to be non-linear to create a limit cycle. The bifurcation boundary of this system can be obtained by assuming exponential time dependence and constructing the following eigenvalue equation [23] where λ is the temporal eigenvalue.

$$|\mathbf{A}-\lambda\mathbf{I}|=0 \tag{5}$$

with $\lambda=0$, the boundary for a pitchfork bifurcation is achieved and for the Hopf bifurcation boundary per se, λ is substituted by $\pm\omega i$ and the above equation will have real and imaginary parts and both of them should be equal to zero. So there are two equations for two unknown parameters a and b for each ω. By solving these equations simultaneously for different values of ω, and plotting the resultant set of a and b, the Hopf bifurcation boundary is determined. On one side of this boundary, the system has a fixed point and the system response settles to zero at large time. On the other side of the boundary, a limit cycle is created and the system will oscillate forever. In this article a set of a and b is considered that creates limit cycle as system response. By comparing the LCO amplitudes of the healthy and damaged systems for different locations along the panel length, the damage extent and its location can be detected.

3. Damage detection of an aeroelastic model using LCO analysis

Yin and Epureanu [23] applied their method on a beam-like panel in a supersonic stream as shown in Fig. 1. The investigated model is essentially a plate with infinite (or large) width, so all of the spatial derivatives with respect to width direction become zero and the structural equation of motion is indeed like a beam. But it is not a beam in a physical sense, because of the flow over the structural system. So it is called a panel to distinguish between the structural and fluid systems. In this paper, exactly same model is investigated and the equation of motion for such system would be

$$DW''''+\rho_m h\ddot{W}-Eh\eta_0 W''-\frac{Eh}{2l}\left[\int_0^1 W'^2(\zeta)\mathrm{d}\zeta\right]W''$$
$$+\frac{\rho_\infty U_\infty^2}{M}\left(W'+\frac{1}{U_\infty}\dot{W}\right)+\Delta p_s-P(X,t)=0 \tag{6}$$

where W' and $\dot{W}$ are the spatial and time derivatives of the panel transverse deflection W, D is the bending stiffness of the panel, ρ_m