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Influence of steady viscous forces on the non-linear behaviour of cantilevered circular cylindrical shells conveying fluid

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ABSTRACT

In this study, the effect of steady viscous forces (skin friction and pressurization) on the non-linear behaviour and stability of cantilevered shells conveying fluid is investigated for the first time. These forces are obtained by using the time-mean Navier–Stokes equations and are modelled as initial loadings on the shell, which are in a membrane-state of equilibrium with in-plane stresses. The unsteady fluid-dynamic forces, associated to shell motions, act as additional loadings on this pre-stressed configuration; they are modelled by means of potential flow theory and obtained by employing the Fourier transform technique. The problem is formulated using the extended Hamilton's principle in which the shell model is geometrically non-linear and based on Flügge's thin shell assumptions. This model includes non-linear terms of mid-surface stretching and the non-linear terms of curvature changes and twist, as well. The displacement components of the shell are expanded by using trigonometric functions for the circumferential direction and the cantilevered beam eigenfunctions for the longitudinal direction. Axisymmetric modes are successfully incorporated into the solution expansion based on a physical approximation. The system is discretized and the resulting coupled non-linear ODEs are integrated numerically, and bifurcation analyses are performed using the AUTO program. Results show that the steady viscous effects diminish the critical flow velocity of flutter and extend the range of flow velocity over which limit cycle responses are stable. On the other hand, the non-linear terms of curvature changes and twist have very little effect on the dynamics. The system exhibits rich post-critical dynamical behaviour and follows a quasiperiodic route to chaos.

1. Introduction

Many biological and engineering systems involve thin-walled shells conveying incompressible fluid flow. Pulmonary passages and veins are examples of these types of structures in physiological systems; heat exchangers, jet pumps and heat shields of jet engines are examples of where such systems may be found in engineering applications.

Given the numerous applications of thin shells conveying fluid, studying their stability and acquiring knowledge about their post-instability behaviour are of great importance in avoiding catastrophic structural failures in engineering systems and in obtaining a better understanding of how biological systems function. A complete account and treatment of this subject may be found in Païdoussis [1, Chapter 7].

Païdoussis and Denise [2] developed the first linear analytical model for clamped–clamped and cantilevered (clamped-free) shells conveying inviscid incompressible fluid, and they also performed experiments. They showed for the first time that at sufficiently high flow velocities shells lose stability in either beam or shell modes: cantilevered shells lose stability by flutter, while supported-end shells do so by divergence (buckling). Since then, a large body of research has been devoted to the study of the linear stability of thin shells subjected to inviscid subsonic flows. Weaver and Unny [3] and Shayo and Ellen [4] investigated the stability of shells with simply-supported ends. Later, Shayo and Ellen [5] studied the linear dynamics of cantilevered shells conveying fluid, recognizing the importance of fluid behaviour beyond the free end of the shell; they introduced the concept of a "downstream flow model".

In all aforementioned studies the fluid viscosity is neglected, and fluid-dynamic forces are modelled assuming that the flow is inviscid. However, in reality fluids are viscous to a greater or lesser extent, and it is of interest to take the fluid viscosity into account and study its effects.

Païdoussis et al. [6] used the time-mean Navier–Stokes equations in conjunction with a linear shell model with clamped–clamped boundary conditions to study the primary aspects of the fluid viscosity, namely the steady-state pressurization (to overcome pressure drop) and the skin frictional force. These forces

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were found to be important as far as stability of the system is concerned. The effect of steady viscous forces on the linear stability of cantilevered shells subjected to either internal or annular flow was analysed by Païdoussis et al. [7]. They found that, in the case of internal flow, these forces have slight stabilizing effect, which becomes more pronounced for annular flow. Nguyen et al. [8] studied the effect of unsteady viscous forces on the linear stability of coaxial cantilevered shells conveying fluid in the annulus by means of a CFD-based model; they found that the unsteady effects of viscosity diminish with decreasing annular gap size, provided that the gap is small enough. Amabili and Garziera [9] studied the effect of steady viscous forces on the linear vibrations of simply-supported shells with non-uniform constraints, lying on an elastic foundation with added masses.

All the aforementioned theoretical work was done by means of linear theory. The linear theory, however, is limited to predicting the dynamics only up to the first loss of stability and is not capable of predicting the post-instability behaviour of the system. Non-linearities come into play at deformation amplitudes of the order of the shell thickness, which in many cases are not hard to achieve. Thus, investigation of the non-linear behaviour of the system is of value from a design point of view.

Investigation of the non-linear behaviour of simply-supported shells conveying inviscid fluid flow has been carried out by Lakis and Laveau [10]. They considered only the non-linearities associated with the fluid flow and found that these non-linearities do not have a considerable effect on the oscillations of the order of the shell thickness. Amabili et al. [11] re-examined the non-linear dynamics and stability of simply-supported shells conveying inviscid fluid. They used Donnell's non-linear shallow-shell theory for the structure and linearized potential flow theory for the fluid. It was shown that the system loses stability through a subcritical divergence. Also, the response of simply-supported shells containing quiescent or flowing fluid under harmonic excitation has been addressed in [12]. Non-linear dynamics and stability of clamped–clamped shells subject to annular or internal inviscid fluid flow was investigated theoretically and experimentally in [13] for the first time. They employed linearized potential flow theory to formulate the fluid-dynamic forces and Donnell's non-linear shallow-shell theory for the structure; comprehensive experiments were also performed. Both theory and experiments show that the system loses stability via a subcritical divergence. Amabili et al. [14] took into account the steady viscous effects in their non-linear model for supported-end shells conveying fluid.

Paak et al. [15] investigated the non-linear stability and the post-instability response of cantilevered shells conveying inviscid fluid for the first time. They found that the system loses stability by flutter in a supercritical fashion (*i.e.*, a supercritical Hopf bifurcation); multiple branches of stable limit cycles were found. The amplitudes of these limit cycles grow with flow velocity until they lose stability and the response becomes non-periodic (*e.g.*, quasiperiodic or chaotic).

One difficulty arising when dealing with shells conveying fluid is the inclusion of axisymmetric modes in the solution expansion. For supported-end shells, this issue is circumvented by assuming an infinite shell with periodic supports and presuming that the shell deformation over $[L, 2L]$ is the reflection of that over $[0, L]$, L being the shell length [13]. Such an artifice is not possible for cantilevered shells conveying fluid. However, in this paper, the axisymmetric modes are included in the solution expansion by using a physically sound approximation.

The influence of steady viscous forces on the non-linear stability and the post-instability dynamics of cantilevered shells conveying fluid had not been explored up to now. In the present study, we extend the theory and investigate the effect of such forces on the non-linear behaviour of the system.

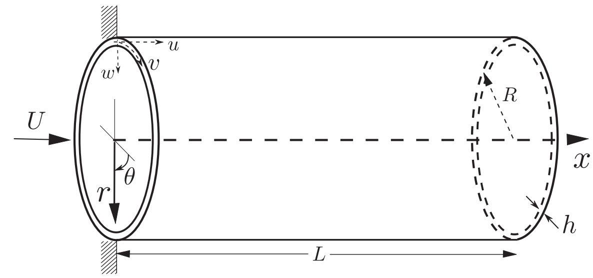

Fig. 1. Schematic of the system.

2. Formulation

2.1. Definitions and assumptions

Fig. 1 is a schematic diagram of the system under consideration. The shell has thickness h, length L, mean radius R and is assumed to be thin (*i.e.*, $h/R \ll 1$), clamped at $x=0$ and free (unsupported) at $x=L$. The displacement components of the shell middle surface along the axial, circumferential and radial directions are denoted by u, v and w, which are functions of time and the middle surface coordinates of the shell (x,y) in which $y=R\theta$. The shell material is considered to be linearly elastic, homogeneous and isotropic, with Young's modulus E, Poisson ratio ν_s and density ρ_s. Viscous damping with coefficient c is considered to model the structural energy dissipation. The shell non-linearity is of geometric type, which is described by large deformation theory.

The fluid is assumed to be incompressible with density ρ_f, flowing in the positive x-direction with a constant mean velocity U.

The unsteady fluid-dynamic forces due to the shell motions are derived by assuming that the fluid flow is inviscid and irrotational, thus enabling the utilization of potential flow theory. These forces are in the radial direction. The pressurization and the skin frictional forces (steady viscous forces) are taken into account as additional radial and axial loadings related to the steady mean flow. This is a simplification which renders the two parts (*i.e.*, the mean flow and perturbation flow fields) decoupled from the outset.

In this paper, partial derivatives may be represented by subscripts preceded by a comma, *e.g.*, $\partial^2()/\partial x\partial y \equiv ()_{,xy}$, and the prime denotes differentiation with respect to the argument of the primed function.

2.2. Equations of motion

The extended Hamilton's principle is utilized to obtain the governing equations of the system; *i.e.*,

$$\int_{t_1}^{t_2} [(\delta\mathcal{T} - \delta\mathcal{U} + \delta\mathcal{W}_d) + \delta\mathcal{W}_f]\, \mathrm{d}t = 0, \tag{1}$$

where δ is the variational operator, $\mathcal{T}$ the shell kinetic energy, $\mathcal{U}$ the elastic strain energy, $\delta\mathcal{W}_d$ the virtual work done by structural damping forces, and $\delta\mathcal{W}_f$ the virtual work done by the unsteady fluid-dynamic forces.

The non-linear shell model is developed based on Flügge's thin shell theory [16] and contains the non-linear terms due to mid-surface stretching as well as the non-linear terms of curvature changes and twist. To be able to perform the calculations in the undeformed reference configuration, Green's strain tensor and the second Piola–Kirchhoff stress tensor (a work-conjugate pair) are used.