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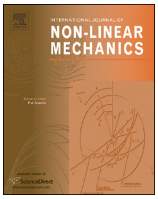

Sensitivity in the structural behavior of shallow arches

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ABSTRACT

It is well established that certain structural buckling problems are extremely sensitive to small changes in configuration: geometric imperfections, load application, symmetry, boundary conditions, etc. This paper considers the behavior of a very shallow arch under lateral point loading, and specifically under the influence of changes in the thermal environment. In some ways the system under study is especially sensitive since small changes influence whether the arch 'snaps-through' or not. The experimental results provide insight into the challenges of understanding the behavior of these types of structural components in a practical, and thus necessarily imperfect, situation. The focus is on static loading or at least quasi-static loading, in which loading occurs on a slow time scale. This study also acts as a backdrop for studying the dynamic behavior of shallow arches, an area of concern in the context of aerospace structural components.

1. Introduction

The shallow arch provides an important paradigm in non-linear structural behavior. It is also quite representative of a broad class of curved, slender members used throughout aerospace structures. Although the geometry of a very shallow arch is not particularly different from that of a flat beam, the response to transverse loading may be quite different. The presence of even small amounts of curvature allows for strongly non-linear behavior including *snap-through* buckling, i.e., the sudden dynamic jump from one equilibrium configuration to a remote (co-existing) configuration that is often associated with an inverted position. Furthermore, the geometric 'depth' of the arch, measured in terms of curvature or the rise/span ratio, may lead to an asymmetric loss of stability, even in those cases where the structure and loading are nominally symmetric.

This paper will focus attention on a specific arch: a thin steel strip whose (unstressed) equilibrium configuration includes a slight curvature, clamped at both ends (i.e., zero displacement and velocity boundary conditions for nodes on each end), subject to transverse point loading, and also under elevated thermal loading conditions. The focus of the research was to assess the structural response relative to imperfections in shape, clamping force, load location, and thermal conditions, i.e., a sensitivity study.

The motivation for this paper arose from experiences at the Air Force Research Laboratory (AFRL) at Wright-Patterson Air Force Base (WPAFB) in Dayton, Ohio, where shallow arches had been the subject of various tests mostly focused on their dynamic response, for example, modal analysis to extract natural frequencies and mode shapes, and a statistical analysis of persistent and intermittent dynamic snap-through behavior of the beam subject to high-frequency excitation. The sensitivity and inherent difficulty associated with repeatability of the load-deflection behavior of these arches had been noted despite the relatively careful testing conditions. This paper will explore the sensitivity of such structures within an (inevitably imperfect or noisy) experimental context. These results will also be discussed in relation to a finite element (FEA) study in which an attempt is made to match the behavior observed in the laboratory.

By way of introduction, consider a shallow, clamped arch as shown in Fig. 1(a). When subject to a point load located at the center of the span, the arch deflects and may snap-through. If the arch is sufficiently shallow (the sketch has an exaggerated vertical scale for clarity), with symmetric loading and boundary conditions, the deflection will typically be symmetric, and on removal of the load the arch may or may not return to its original configuration. Fig. 1(c) shows a representative set of data taken from the lab, used here for illustrative purposes: details related to the systematic experimental study will be given later. The force, F, was applied via a displacement-controlled device that *pushed* on the arch. The force (measured by a load cell) increased and then decreased until losing contact as the arch suddenly snapped to its inverted configuration, which is indicated by the lowest dashed curve in part (a). The loading mechanism was then re-contacted and further data

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were taken, and a reversal in the direction of loading (shown in red) until contact was lost again. This behavior is typical of shallow arches.

Under *dead loading*, in which a force is due to a weight in gravity, for example, the system would have snapped at the maximum of the load-deflection curve; a horizontal tangency corresponds to a loss of stiffness. Under *displacement-controlled loading* the complete load-deflection curve would have been traced-out provided the geometry is such that no vertical tangency is encountered. The loading scenario described above (displacement-controlled but positive force only) is somewhat intermediate between these two and results in the observation of a partial snap, i.e., a jump from a point that does not necessarily correspond to a turning point in the load-deflection curve.

However, a few days later it was decided to repeat the experiment. There were no obvious changes to the system between experiments. The data from this later experiment are shown in Fig. 1(d). The load-deflection behavior changed in a number of ways. For example, the maximum load decreased slightly, as did the deflection at which the maximum force occurred. However, the most significant and easily observed difference between these two scenarios is that in the latter case the arch did *not* exhibit (the abbreviated) snap-through, with a continuous (positive force) 'push' on the load cell throughout the measured range. This paper is aimed at explaining this unexpected behavior, and assessing the likely cause(s). Fig. 1(b) shows a typical schematic load-deflection relation associated with a non-linear equilibrium path. This type of behavior is not uncommon in many branches of science and engineering, with the appearance of a pair of saddle-node bifurcations together with hysteresis. It is clear that the behavior must be related to a change in the 'unloaded' configuration. This can be viewed as a kind of prestress (F_1 and F_2), and is likely sensitive to initial geometry, clamping force, or even thermal environment. In fact, it is also likely that the form of the curve might change as well, with a third possibility corresponding to a curve that would be single-valued in force, eliminating the possibility of snap-through under any loading conditions. These results are consistent with the differences exhibited by arches with different 'depths', i.e., central height-to-span ratios. Thus, we seek to isolate cause-and-effect factors that contribute to these seemingly small changes, but accounting for the relatively large effect on snap-through behavior.

A schematic of a possible scenario is shown in Fig. 2, where we basically have a pair of saddle-node bifurcations forming a region of hysteresis. The vertical axis labeled T is associated with temperature and the fundamental effect is to increase the rise of the arch. The horizontal axis represents transverse deflection, at the arch center. The blue dot indicates where the birth of

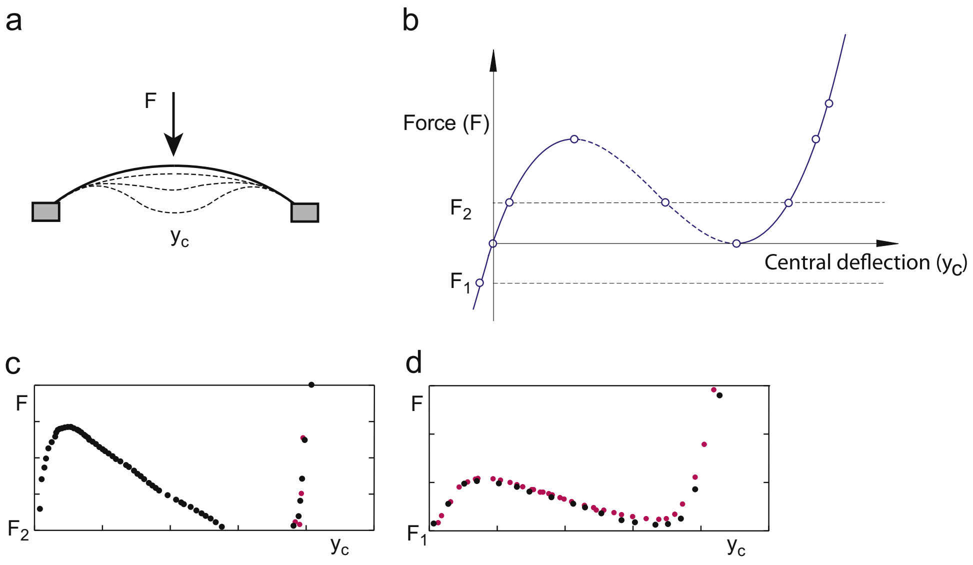

Fig. 1. (a) A centrally loaded arch, (b) force–displacement relation with two different zero-force datums, (c) representative force-deflection data, (d) representative force–displacement data taken at a later time. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

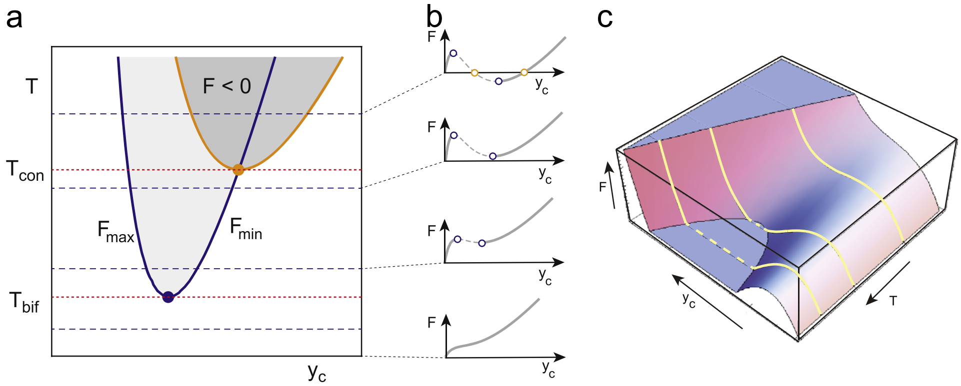

Fig. 2. A schematic of how the number of turning points and shifting of the equilibrium configurations could be influenced by the thermal environment. (a) The appearance of turning points in the equilibrium paths as a function of temperature, (b) some specific equilibrium paths, (c) a surface plot showing the relation between load, deflection and temperature. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)