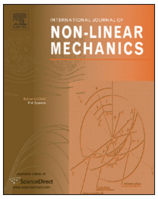
International Journal of Non-Linear Mechanics 57 (2013) 39–49

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm



An analytical model for stress analysis of short fiber composites in power law creep matrix

M. Mondali [a,*], A. Abedian [b]

[a] Department of Mechanical and Aerospace Engineering, Islamic Azad University, Science and Research Branch, Tehran, Iran
[b] Department of Aerospace Engineering, Sharif University of Technology, P.O. Box 11365-8639, Azadi Avenue, Tehran, Iran

ARTICLE INFO

Article history:
Received 23 July 2012
Received in revised form
27 May 2013
Accepted 8 June 2013
Available online 18 June 2013

Keywords:
Short fiber composites
Steady state creep
Stress analysis
Finite element modeling
Geometric factors

ABSTRACT

The creep deformation behavior of short fiber composites has been studied by an approximate analytical model. A perfect fiber/matrix interfacial bond is assumed and a power law function is considered for describing the steady state creep behavior of the matrix material. The results obtained from the proposed analytical solution satisfy the equilibrium and constitutive creep equations. Also, a parametric study was undertaken to define the effects of geometric parameters on the steady state creep strain rate of short fiber composites. The present model is then validated using the results of finite element method. The predicted strain rate and stress components by the proposed analytical approach exhibit good agreement with the finite element results.

© 2013 Elsevier Ltd. All rights reserved.

1. Introduction

In recent years, the high-temperature creep behavior of short fiber composites has become a topic of considerable interest, primarily because these materials have a high potential for use in structural applications at elevated temperatures [1–11]. Kelly and Tyson [1,2] have used a frictional energy dissipation approach for strength behavior in metal matrix composites. They showed that the continuous fiber reinforced metal matrix composites display significantly better creep resistance and stress rupture life than the corresponding unreinforced matrix material [1–3]. Tension creep tests of 6061Al/20% SiC whisker composites have been conducted in the temperature range 505–644 K by Nieh [4] in 1984. He reported that the stress exponent for the steady state creep rate of the composite is approximately 20.5 and is not essentially a function of temperature while the stress exponent of the matrix material is 4. Nardone and Strife [5] examined the effects of stress and temperature on the creep behavior of SiC whisker reinforced 2124Al composite. They showed that the stress exponent changes from 8.4 at 450 K to 21 at 561 K. Dragon et al. [6] have investigated the creep behavior of other kind of metal matrix composites such as lead reinforced with nickel fibers. A review of creep data of discontinuous SiC–Al composites has been conducted by Mohamed et al. [7] which shows that the value of the stress exponent is high and also variable. The conventional high temperature creep rupture tests on a commercially produced metal matrix composite and a wrought unreinforced equivalent alloy were performed by Greasly [8]. He reported that at typical creep service loads the performance of the reinforced alloy is superior. Also, some researchers have studied the mechanisms of longitudinal creep deformation and damage in laminated composites [9]. Ma and Tjong [10] have investigated the tensile creep behavior of SiC-whisker and particle reinforced 2124Al matrix composites and unreinforced 2124Al alloy at 573–673 K. Their results exhibited that the unreinforced 2124Al alloy has the stress exponents of 4.7–8.5 while the particulate and whisker-reinforced composites have the stress exponents of 9.4–17.1 and 11.5–18.2, respectively. Mondali et al. [11] highlighted the shortcomings of the previous FEM studies and combines the analytical findings with the power of FEM modeling. More specifically, they incorporated the fiber/matrix debonding parameter into the modeling and showed that in contrary to the available analytical results, the value of debonding parameter could only change in the range of 0–0.5.

The increasing number of applications of short fiber composites makes it more important to understand and predict their creep characteristics and deformation mechanisms. It is well known that short fiber composites display significantly better creep resistance than the corresponding unreinforced matrix alloy. As it will be fully discussed in following sections, the unit cell of a short fiber composite can be divided into two separate regions which are considered as two continuous fiber composite models. Therefore,

---

[*] Corresponding author. Tel./fax: +98 21 44865239.
E-mail address: mondali@srbiau.ac.ir (M. Mondali).

0020-7462/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ijnonlinmec.2013.06.007

a proper formulation for continuous or long fiber composite models should be derived, first. Note that in comparison with short fiber composites, creep modeling of continuous or long fiber composites is simpler because of the end stress of fibers in short fiber composites can hardly be determined. In recent years, some researchers have tried to analyze the short fiber composites using a new developed shear lag model namely imaginary fiber technique which is only presented for elastic problems [12–20].

A variety of analytical and numerical models have been developed to predict the second stage creep strain rate of short fiber composites [11,21–31]. These models mainly include the shear lag models [21–30], and the numerical models based on the finite element method (FEM) [11,31]. Compared with the numerical models, the shear lag model is simplest mathematically. However, because of simple assumption of the matrix uniform deformation rate, it cannot describe the variation of the stress distributions in the matrix and the fiber during the steady state creep deformation. Except for stress transfer from the matrix to the fiber, the shear lag model cannot well predict the composite creep strain rate. Therefore, the application of the shear lag model to short fiber composites has become limited over the time.

In general, solution of the analytical model involves with two equilibrium equations and four constitutive equations [32]. In the shear lag models, only one of the equilibrium equations is used to solve the problem. Thus, the average axial stress in the fiber and matrix and the interfacial shear stress (i.e. $\overline{\sigma}_z^f$, $\overline{\sigma}_z^m$, and τ_i) as a function of fiber axial direction (z) could be obtained. In fact, instead of the constitutive equations, the simple velocity relations of the flowing matrix are used for defining the creep strain rate of the composite. Therefore, the shear-lag model cannot predict the stress components and also the displacement rates of the composite as functions of radial (r) and axial (z) coordinates.

The above discussions show that a relatively simple analytical solution is still highly desired for describing the steady state creep behavior of short fiber composites. For such an approximate analytical solution the stress field components in the matrix and the fiber should satisfy all the constitutive and equilibrium equations considering the imposed boundary conditions. In general, with the axisymmetric assumption for problem geometry, the equilibrium and constitutive equations cannot be exactly satisfied even for the composites with elastic behavior of the matrix. However, some researchers have solved the elastic problem considering various approximations and simplifications made in the shear lag model [12–18]. Therefore, developing an analytical solution for creep behavior in short fiber composites is very complicated and should be obtained by FEM or other numerical methods.

In 2009, Mondali et al. [33] developed a new analytical model based on the shear-lag theory for stress analysis and prediction of the steady state creep deformation in short fiber composites. In this new analytical model, a perfect bonding at the fiber/matrix interface was assumed and the steady state creep behavior of the matrix was described by an exponential law. Note that generally the creep behavior of the most metals and alloys at high temperatures is described by a power law. Consequently, some experiments data obtained for the metal matrix composites with matrix creeping are analyzed on the basis of the power law [25]. Hence, it is very essential to develop a new analytical solution for defining the second stage creep strain rate of short fiber composites when the steady state creep behavior of the matrix has been described by a power law. The main objective of the present study is thus to develop such an analytical model for stress analysis of short fiber composites in power law creep matrix. This model should satisfy the constitutive, and equilibrium equations and the overall boundary conditions. Since the geometrical shape and arrangement of fibers have a strong effect on the creep strain rate of short fiber composites, a parametric study is undertaken to study the effects of these factors on the steady state creep strain rate of short fiber composites. Here, an axisymmetric unit cell representing a fiber with its surrounding matrix as two coaxial cylinders is assumed. For verification of the solution method, the Al6061/20%SiC composite [31] is selected as a case study and the results will be compared with the FEM results.

2. Problem definition

2.1. Composite model

The cylindrical unit cell shown in Fig. 1 has been used by many researchers [12–18,25–27,29] to model a short fiber composite. In this model a cylindrical fiber with a radius a and a length $2l$ is surrounded by a coaxial cylindrical matrix with an outer radius b and a length $2l'$. The volume fraction and aspect ratio of the fiber are defined as $f=(a^2 l)/(b^2 l')$ and $s=l/a$, respectively. Also, $k=l'a/lb$ is a parameter related to the geometry of the unit cell. Axial stress σ_0 is uniformly applied on the end faces of the unit cell, i.e. $z=\pm l'$. The cylindrical coordinate system (r,θ,z) is used with the origin shown in Fig.1. Due to symmetry in geometry, loading, and boundary conditions, the analysis is performed only on half of the unit cell, i.e. $0 \le z \le l'$.

In this analysis, the following assumptions are made:

(i) Steady state condition of stress is assumed.
(ii) Elastic deformations are small and negligible as compared to creep deformations.
(iii) The fibers behave elastically during the analysis and the steady state creep behavior of the matrix is described by a power law as given in Eq. (1),

$$\dot{\varepsilon}_e = A\sigma_e^n \tag{1}$$

(iv) Perfect bonded at fiber/matrix interface is considered.

2.2. Equilibrium and constitutive equations

The governing equilibrium equations for the axisymmetric problem considering the cylindrical coordinates (r,θ,z) are obtained as

$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{rz}}{\partial r} + \frac{\tau_{rz}}{r} = 0 \tag{2a}$$

$$\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \tag{2b}$$

The generalized constitutive equations and the strain rate-displacement rate relations for creep deformation of the matrix material in r, θ and z directions [32,33] are

$$\dot{\varepsilon}_r = \frac{\partial \dot{u}}{\partial r} = \frac{\dot{\varepsilon}_e}{2\sigma_e}[2\sigma_r - \sigma_\theta - \sigma_z] \tag{3a}$$

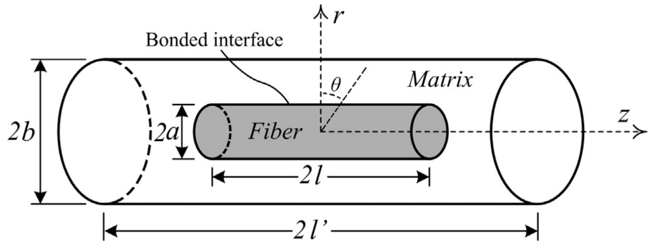


Fig. 1. Representation of the unit cell.

Download English Version:

https://daneshyari.com/en/article/7174671

Download Persian Version:

https://daneshyari.com/article/7174671

Daneshyari.com