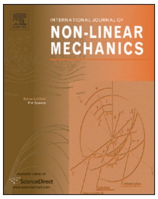
Contents lists available at SciVerse ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm



Stability of multi-column systems with initial imperfections and non-linear connections

Q1 J. Dario Aristizabal-Ochoa *

School of Mines, National University, Medellín, Colombia

ARTICLE INFO

Article history:
Received 11 April 2013
Received in revised form
20 June 2013
Accepted 21 June 2013

Keywords:
Columns
Frames
Inelastic behavior
Imperfections
Second-order analysis
Stability

ABSTRACT

The stability and second-order response of an unbraced multi-column system with initial geometric imperfections (i.e., made of columns with initial curvature, out-of-plumb in the plane of bending under eccentric axial loads at both ends), and non-linear end connections subjected to a lateral load at the top floor level are presented in a classical manner. The proposed method is based on the Euler–Bernoulli theory and limited to 2D multi-column systems with sidesway uninhibited or partially inhibited by lateral bracing at the top level. The combined effects of initial imperfections and non-linear connections in the plane of bending are condensed into the proposed governing equations which also can be used to evaluate the induced moments and second-order deflections along each column as the lateral and axial loads are applied to each column at the top level. The non-linear end connections are modeled using the three-parameter model suggested by Richard and Abbott. The bracing force is defined as a function (linear or non-linear) of the lateral sway. Three comprehensive examples are presented in detail that demonstrate the effectiveness of the proposed method and show the combined effects of initial imperfections and non-linear connections on the stability and second-order response of 2D multi-column systems. The proposed method can be used by structural researchers and engineers to investigate these effects on the stability and second-order response of multi-column systems including cases with relatively large lateral sways.

1. Introduction

The inelastic stability and second-order analyses of framed structures are of great importance in structural engineering, particularly those with slender columns subjected to heavy vertical and lateral loads. However, these analyses in the inelastic range are extremely cumbersome requiring the use of complex non-linear Finite Element Method (FEM) algorithms.

In multi-column systems the lateral deflections along each column (so called "bowing" or P–δ effects), the inter-story drift (P–Δ effects), the initial geometric imperfections of the columns (like initial curvature, out-of-plumb, and eccentric axial loads), and end connections cause non-linear behavior with additional bending moments, rotations and displacements. These effects which are amplified by the applied axial forces alter the stiffness and reduce the buckling capacity of each column and also those of the structure as a whole. They can lead to partial or total collapse, particularly when the inter-story drift is uninhibited and the rotational restraints at both ends of the columns and lateral bracing are non-linear with low stiffness, limited ductility and strength. In addition, in multi-column systems columns are coupled together by the connecting floor and their behavior under vertical and lateral loads are affected by many factors including their sectional and material properties and slenderness, the moment-rotation characteristics of the end connections, residual stresses, initial geometric imperfections, and the load-deflection characteristics and behavior of the lateral bracing. Since structural components are geometrically imperfect, real frames are also imperfect, consequently lateral deflection commences as soon as loading is applied. In real design and analysis of framed structures, the effects of geometric imperfections must be taken into account right from the start. The most important initial geometric imperfections in columns are the initial curvature (or out-of-straightness described as a lateral deflection of each member relative to its cord), the out-of-plumb (or initial lateral sway described as the lateral displacement of one end of each column relative to the other); and the eccentricities of the applied end loads.

Razzaq and Calash [1] carried out an analytical study on the effects of biaxial semirigid connections on the inelastic response of hollow rectangular steel non-sway columns with biaxial crookedness and residual stresses. They found that residual stresses were less detrimental to column strength than initial crookedness, the strength of a crooked column increases with the degree of fixity of its end connections, and that the column maximum axial load

---

Q2 * Tel.: +57 42686218; fax: +57 44255152.
E-mail addresses: jdaristi2@yahoo.com, jdaristi@unal.edu.co

0020-7462/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ijnonlinmec.2013.06.012

Please cite this article as: J.D. Aristizabal-Ochoa, Stability of multi-column systems with initial imperfections and non-linear connections, International Journal of Non-Linear Mechanics (2013), http://dx.doi.org/10.1016/j.ijnonlinmec.2013.06.012

Nomenclature

A	gross area of column cross-section;
a	initial camber at midspan of column;
a_n	peak value of the initial camber corresponding to n-wave;
E	elastic modulus;
e_a and e_b	axial load eccentricities at ends A and B, respectively;
F	applied horizontal force at the top B;
G_{ai} and G_{bi}	traces on the $M_{ai}M_{bi}$ plane defined by Eq. (10a) and (10b), respectively;
h	column height;
I	moment of inertia of column cross-section in the plane of bending;
M_u	ultimate moment capacity of end connection;
$M_a^* = M_a + Pe_a$ and $M_b^* = M_b - Pe_b$	induced end moments in the non-linear end connections at A and B, respectively [+ clockwise when applied by the end connection to the column]
P	applied axial load at both ends of column;
P_e	$\pi^2 EI/h^2$;
n	shape parameter of the end connection;
S	stiffness of the lateral elastic bracing at the top end B;
u	additional second-order lateral deflection along the column height caused by the applied axial load P;
u_o	initial column imperfection defined as $(x\Delta_o/h)$;
u_1	initial column imperfection (curvature) defined by a parabola symmetric about column midspan or by a series of sinusoidal waves;
$u_T = u + u_o + u_1$	total transverse deflection;
Δ	sway of column's top end with respect to a vertical line at end A;
Δ_o	initial out-of-plumb of the column;
Δ^*	$\Delta - \Delta_o =$ net lateral sway of the column's top end;
ϕ	$\sqrt{Ph^2/(EI)}$;
κ_a and κ_b	an initial stiffness of the lumped bending non-linear connections at ends A and B, respectively;
ψ_a^* and ψ_b^*	rotations of the non-linear end connections caused by the induced moments M_a^* and M_b^*, respectively.

capacity is always associated with a severe or complete plastification at and around the midspan. Papadrakakis and Loukakis [2] studied analytically the inelastic behavior of a prismatic column subjected to cyclic loading including the effects of an initial out-of-straightness, eccentricity of the applied axial load, and end restraints taking into consideration the plastic flow in the vicinity of hinging regions. Yau and Chan [3] developed a method to trace the equilibrium path of steel frames, allowing for geometrical, material and non-linear connections up to the ultimate load using a beam-column element with springs connected in series. King and Chen [4] studied the non-linear stability of rigid and semirigid frames bent about the weak axis using a hardening plastic hinge method.

Chan and Zhou [5] presented a method that includes the effects of initial imperfection on the column element stiffness matrix. Kim and Chen [6,7] presented procedures for 2D braced and unbraced steel frame analysis and design including gradual yielding associated with flexure, residual stresses, second-order effects and geometric imperfections. Chan et al. [8] developed a finite element procedure for large-deflection and inelastic analysis of imperfect steel frames with semirigid bases with various modes of initial imperfection. Surovek et al. [9] presented an approach that allows for the consideration of non-linear connection response in framed structures with semirigid connections using commonly available elastic analysis software.

Eröz et al. [10] studied the effects of partial base fixity using the direct analysis method (DM) promoted by the 2005 AISC specification for the stability design of steel frames. Xu and Wang [11] carried out parametric studies on the effects of initial imperfection and out-of-plumb on the lateral stability of unbraced plane frames. They found that(1) the effective length factor increases almost linearly with the initial geometrical imperfections; (2) the out-of-straightness has a greater detrimental effect than that of the out-of-plumb; and (3) given the same value of initial geometric imperfection, the influence of the out-of-straightness on the column effective length factor is almost doubled as that of the out-of-plumb. They concluded that "…This finding is consistent with current practice in which the tolerance for the out-of-straightness and the out-of-plumb are $L/1000$ and $L/500$, respectively."

The elastic stability analysis of perfect 2D and 3D multi-column systems has been also investigated previously by Aristizabal-Ochoa [12–19] showing that the lateral stability and second-order response of 2D and 3D multi-column systems are very sensitive to the provided lateral bracing, the height and stiffness of each column, the plan layout (location and orientation), stiffness of the end connections, and axial load distribution among the columns. Some of these features are neither mentioned in the technical literature nor are addressed by the current construction codes and should be taken into account in the analysis and design of multi-column systems. Fallacies and misconceptions in the stability analysis and classification of columns in current construction codes were also presented and discussed in Refs. [15–17].

The slope-deflection equations for the second-order analysis of plane framed structures made of members with elastic semirigid connections, initial imperfections and out-of-plumb subject to static loads have also been developed by the author [20]. It is shown that initial imperfections in beams and columns act as they were subjected to additional transverse loads proportional to the bending stiffness and magnitudes of the imperfections increasing their lateral deflections, moments, and shears. Recently, closed form equations for the induced end moments, shear forces and lateral deflections in columns and plane frames with elastic semirigid connections and imperfections subjected to eccentric axial loads have been presented by the author [21–23], and more recently the effects of non-linear connections have been included in Ref. [24].

Studies on the combined effects of initial imperfections and non-linear connections on the stability and second-order response of multi-column systems are still not available in the technical literature. The main objective of this publication is precisely that, to present an analytical approach and the corresponding equations that can be used to evaluate the stability and second-order lateral response of 2D multi-column systems with initial geometric imperfections (i.e., columns with initial curvature, out-of-plumb in the plane of bending and axial load eccentricities), non-linear connections, and lateral bracing subjected to eccentric axial loads at both ends of each column and a lateral load at the top floor level. The proposed method is (1) based on the classical Euler–Bernoulli theory and applicable to 2D multi-column systems with sidesway uninhibited or partially inhibited and (2) limited to systems subject to vertical gravity loads and to lateral load at the top floor causing bending about one of the principal axes of each column, with the inelastic behavior taking place at the end rotational connections of the columns. The effects of torsion, shear

Please cite this article as: J.D. Aristizabal-Ochoa, Stability of multi-column systems with initial imperfections and non-linear connections, International Journal of Non-Linear Mechanics (2013), http://dx.doi.org/10.1016/j.ijnonlinmec.2013.06.012