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Inhomogeneous deformations and pull-in instability in electroactive polymeric films

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ABSTRACT

By means of a bifurcation analysis we show the onset of inhomogeneous equilibrium configurations in thin electroelastic polymeric films under assigned voltage. The resulting activation threshold decreases the diffusely adopted value obtained under the assumption of homogeneous deformations. We argue that the bifurcated inhomogeneous solution describes experimentally observed localization effects.

1. Introduction

Actuation devices based on Electroactive Polymers (EAP) are typically constituted by thin flat polymeric films, sandwiched between compliant electrodes [7]. When a voltage is applied, the Coulomb forces induce an in-plane expansion used as a mean of actuation (see Fig. 1).

When the voltage difference is increased, the balance between the electric forces and the elastic forces of the polymeric film may become impossible with the system undergoing the so-called *pull-in* instability, one of the main failure effects observed in EAP devices. The first clear experimental evidence of such instability can be traced back to the work by Stark and Garton [33] where the loss of equilibrium was described by means of a 'toy model'. The growing development of technological devices based on EAP actuators has recently raised a broad interest in the instability phenomena of electro-activated soft materials and in particular in the prediction of pull-in instability.

The most effective models of this phenomenon are typically based on a variational approach, where the stability is analyzed by imposing stationarity conditions of the potential electromechanical energy (see [5,35], and references therein) or positive definiteness conditions of the energy function [17]. An important hypothesis generally adopted in these approaches is the restriction to the case of homogeneously deformed films. This simplifying assumption has been at the base of a clear analytic description of the role of some important aspects regarding boundary conditions, prestretch, and entropic hardening of the polymers [12,13,37]. Nevertheless, localization and inhomogeneity effects are generally observed, as documented since the pioneering work [4] where the authors remarked that the deformation localization may explain "the poor agreement between the observed dielectric strength and that predicted theoretically". As in typical cases of discharging phenomena, the onset of instability is due to curvature and inhomogeneity effects in the postcritical configuration, inducing charge localization and, hence, higher localized electric fields [14,29]. Thus, an effective analysis of the phenomenon requires the consideration of inhomogeneous deformations.

Important exceptions to the assumption of homogeneous equilibrium states can be found in the following works. The onset of inhomogeneous deformations was modeled under the assumption of non-convex elastic energy densities in [39] in relation to recent experimental observations [26,28]. In [11] the authors showed that the observed localizations can be explained as a result of compression-induced wrinkling instabilities, analyzed in the spirit of the so called Tension Field Theory for membranes in [12,13]. Also, inhomogeneity effects were considered in [18] where the authors studied surface instabilities induced by inhomogeneous perturbations of an electroelastic half-plane under a uniform assigned electric field. In [18], differently from the case considered in this paper, this instability

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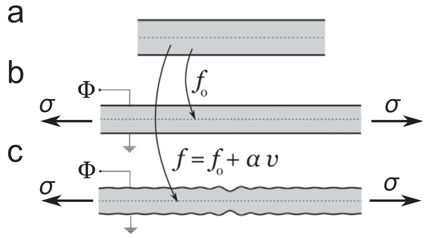

Fig. 1. Scheme of an EAP actuation device under a given voltage difference Φ between the electrodes and a traction σ on the lateral edge. The scheme shows the transition from a homogeneously deformed film (b) to a non-homogeneous configuration (c). As we show in the paper this transition may anticipate the pull-in instability.

is a result of an assumption of a *constitutive* electromechanical coupling. Then, in the case of elastic dielectric composites, instability effects induced by inhomogeneous deformations have been recently considered in [2]. Also, the onset of inhomogeneous deformations for dielectric membranes with non-convex electro-mechanical energies was numerically shown in [40] and a more efficient description of the post-instability configurations was then obtained through a dynamic FEM method in the recent paper [25]. Creasing instabilities for a polymer layer constrained on a rigid electrode were experimentally evidenced in [37] and theoretically explained by a scaling analysis. Finally an interesting theoretical analysis of a surface instability in soft films under surface interactions has been carried on in [32] where the authors perform a bifurcation analysis for a nearly incompressible, linearly elastic solid film attached to a rigid contactor.

Here we propose a variational approach based on the total potential energy of the EAP film that depends on the purely elastic energy, on the total charge on the conductors and on the mechanical external work. Based on the recent results in [29], delivering an analytical solution of the electric field for "small deformations" superimposed to finite homogeneous deformations, we obtain an explicit expression of the total potential energy depending only on the mechanical fields. By means of a bifurcation analysis, we then obtain explicit bifurcation conditions leading to inhomogeneous states. We then deduce the corresponding bifurcation voltage and strain thresholds and show that the restriction to homogeneous states may lead to an overestimation of the critical voltage and activation properties of the device [7]. It is important to remark that the proposed approach enables us to overcome some known theoretical drawbacks in the analyses based on the introduction of different electro-mechanical stress tensors [6,19]. Finally we believe that our approach may provide an interesting tool in the FEM analysis of post-critical behaviors of the electromechanical EAP devices [25].

Recently, in [14] a similar variational approach in a one-dimensional setting let the authors obtain a fully non-linear equilibrium analysis deducing a significant reduction (up to 30%) of the pull-in thresholds due to inhomogeneity effects.

As a final remark, we speculate that the obtained inhomogeneity effects would be magnified under the assumption of a constitutive electromechanical coupling and in particular of a dependence of the dielectric permittivity on the deformation. Similarly, we argue that the presence of material inhomogeneities (voids, contaminations or defects, heating and the resulting softening, damage or plasticity phenomena) can amplify the inhomogeneity effects analyzed in this paper [7,9,10,22,38].

2. Energy balance and bifurcation analysis

In the following we analyze the electromechanical equilibrium problem for an elastic dielectric thin film, sandwiched between two compliant electrodes under assigned voltage (see Fig. 1). Denote by $\boldsymbol{f}$ the deformation of the dielectric film from a reference configuration $\mathcal{B}_0$ to the current configuration $\mathcal{B}$. The upper and lower surfaces of the thin film are subjected to a potential $\Phi^+ = \Phi$ and $\Phi^- = 0$, respectively. Let then $\mathcal{Q}$ be the total charge on the upper electrode.

In this work, following an energetic approach, we search for the equilibrium configurations as stationary points of the total potential energy functional. We adopt the assumption of ideal, isotropic, dielectric elastomer (see *e.g.* [35]), with the total stored internal energy admitting the additive decomposition

$$\mathcal{E}^{tot} = \mathcal{E} + \tfrac{1}{2}\Phi\mathcal{Q}, \tag{1}$$

where

$$\mathcal{E} = \mathcal{E}(\boldsymbol{f}) = \int_{\mathcal{B}_0} W(\nabla\boldsymbol{f}) \tag{2}$$

is the elastic energy of the body, W is the strain energy density function and $\frac{1}{2}\Phi\mathcal{Q}$ is the total electrostatic energy in the current configuration $\mathcal{B}$.

In the case of assigned voltage (see *e.g.* [20,24]) the total potential energy $\mathcal{G}$ takes the form

$$\mathcal{G} = \mathcal{G}(\boldsymbol{f}, \phi) = \mathcal{E}(\boldsymbol{f}) - \mathcal{P}(\boldsymbol{f}) - \frac{\Phi}{2}\mathcal{Q}(\boldsymbol{f}, \phi), \tag{3}$$

where we have evidenced the field dependence of the different energetic components. Here ϕ is the material description of the electrostatic potential and $\mathcal{P}$ represents the work of the external mechanical (conservative) forces.

To determine the equilibrium solutions we use the following strategy. Under the classical assumption of *instantaneous relaxation to equilibrium* of the electric potential, for a given (smooth) deformation $\boldsymbol{f}$ we denote by ϕ_f the unique solution of the electrostatic problem within the domain $\mathcal{B} = \boldsymbol{f}(\mathcal{B}_0)$, whose existence and uniqueness are ensured by classical results for a system of conductors and linear dielectrics (see *e.g.* Section 33 of [23]).

Thus, it is possible to consider a new energy functional $\tilde{\mathcal{G}}$ depending only on the deformation $\boldsymbol{f}$,

$$\tilde{\mathcal{G}}(\boldsymbol{f}) = \mathcal{G}(\boldsymbol{f}, \phi_f) = \mathcal{E}(\boldsymbol{f}) - \mathcal{P}(\boldsymbol{f}) - \frac{\Phi}{2}\tilde{\mathcal{Q}}(\boldsymbol{f}) \tag{4}$$

where

$$\tilde{\mathcal{Q}}(\boldsymbol{f}) = \mathcal{Q}(\boldsymbol{f}, \phi_f) \tag{5}$$

is the total charge corresponding to ϕ_f. Then the mechanical equilibrium is imposed by solving the stationarity condition

$$D\tilde{\mathcal{G}}(\boldsymbol{f})[\boldsymbol{y}] = \frac{d}{dt}\tilde{\mathcal{G}}(\boldsymbol{f} + t\boldsymbol{y})\Big|_{t=0} = 0 \tag{6}$$

for any variation $\boldsymbol{y} = \boldsymbol{y}(\boldsymbol{X})$.

As we will show in Sections 3 and 6, the possibility of applying this strategy in the present case relies on the determination of the explicit solution ϕ_f for a *small inhomogeneous deformation* superimposed on a finite deformation one.

Consider thus a deformation path from a fundamental homogeneous deformation $\boldsymbol{f}_0$,

$$\boldsymbol{f} = \boldsymbol{f}_0 + \alpha\boldsymbol{v}, \tag{7}$$

where $\alpha \in \mathbb{R}$ is a smallness parameter and $\boldsymbol{v}$ is a displacement field defined on the reference configuration $\mathcal{B}_0$. Thus if we introduce the functions

$$\begin{aligned}
\hat{\boldsymbol{E}}(\alpha, t) &= \mathcal{E}(\boldsymbol{f}_0 + \alpha\boldsymbol{v} + t\boldsymbol{y}),\\
\hat{\boldsymbol{P}}(\alpha, t) &= \mathcal{P}(\boldsymbol{f}_0 + \alpha\boldsymbol{v} + t\boldsymbol{y}),\\
\hat{\boldsymbol{Q}}(\alpha, t) &= \tilde{\mathcal{Q}}(\boldsymbol{f}_0 + \alpha\boldsymbol{v} + t\boldsymbol{y}),\\
\hat{\boldsymbol{G}}(\alpha, t) &= \tilde{\mathcal{G}}(\boldsymbol{f}_0 + \alpha\boldsymbol{v} + t\boldsymbol{y}),
\end{aligned} \tag{8}$$