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The Rayleigh–Lamb wave propagation in dielectric elastomer layers subjected to large deformations

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ABSTRACT

The propagation of waves in soft dielectric elastomer layers is investigated. To this end incremental motions superimposed on homogeneous finite deformations induced by bias electric fields and pre-stretch are determined. First we examine the case of mechanically traction free layer, which is an extension of the Rayleigh–Lamb problem in the purely elastic case. Two other loading configurations are accounted for too. Subsequently, numerical examples for the dispersion relations are evaluated for a dielectric solid governed by an augmented neo-Hookean strain energy. It is found that the phase speeds and frequencies strongly depend on the electric excitation and pre-stretch. These findings lend themselves at the possibility of controlling the propagation velocity as well as filtering particular frequencies with suitable choices of the electric bias field.

1. Introduction

The goal of this work is to investigate the propagation of electromechanical induced waves in a dielectric elastomer (DE) layer subjected to finite deformations. In the purely elastic case the first solution for surface waves based on the exact equations of (2D) elasticity was introduced by Rayleigh [1] in 1887, who determined the so-called Rayleigh waves. This work was later extended for propagation of waves in elastic plates by Rayleigh [2] himself and Lamb [3]. Here we extend the Rayleigh–Lamb problem and account for incremental motions superimposed on finite deformations in dielectric media, and investigate how these are influenced by the presence of external electric field and pre-stretch.

When subjected to an electric field electroactive polymers (EAPs) deform and both their mechanical and electrical properties are modified. In contrast to piezoelectric ceramics, DEs are capable of undergoing large deformations, a property that entitled them the name “artificial muscles”. Moreover, while in piezoelectricity the electromechanical coupling is linear, in DEs the mechanical fields depend quadratically on the electric field. A proper theory, which accounts for the aforementioned coupling and captures the ability of the material to undergo finite strains is therefore required. The foundations of this non-linear electroelastic theory are summarized in the pioneering works of Toupin [4] and Eringen [5] for the static case. These contributions were later extended by Toupin [6] to account for the dynamics of these elastic dielectrics. A comprehensive summary can be found in monographs by Eringen and Maugin [7] and Kovetz [8]. Due to the development of new materials that admit this coupled behavior, thus branching toward a window of new applications (e.g., [9–11]), the interest in these electroelastodynamic theories revived and the coupled electromechanical theory was revisited recently (e.g., [12–15]). The foregoing works and their extension to the domain of soft dielectric composites by deBotton et al. [16] and Bertoldi and Gei [17] differ in their constitutive formulations, the choice of the independent variables, the resultant electrostatic stress-like tensors and electric body-like forces. For a review of the diverse formulations of the constitutive laws and governing equations the reader is referred to Bustamante et al. [18]. Among the various approaches we recall the formulation proposed by Dorfmann and Ogden [13] for the static case, and its extension to dynamics by Dorfmann and Ogden [19]. In these works the concept of ‘total’ stress tensor that is derived from a ‘total’ or ‘augmented’ energy-density function was employed. In this work we follow the framework proposed by the latter.

In contrast with the relatively large body of theoretical works that are available, only a few boundary-value problems (BVP) were solved in the context of the dynamic behavior of electromechanically coupled EAPs. One of the first contributions in the field of piezoelectricity was made by Tiersten [20], who examined the thickness vibrations of an infinite piezoelectric plate induced by alternating voltage at the surface electrodes, and later solved

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the corresponding problem of wave propagation [21]. To the best of the authors knowledge, while for piezoelectric media solutions for BVP accounting for the influence of pre-strain and bias field are available (see review article by Yang and Hu [22]), analogue developments in the context of dielectrics were considered by Mockensturm and Goulbourne [23] and Zhu et al. [24], who examined the dynamic behavior of dielectric elastomer balloons, and by Dorfmann and Ogden [19], who studied the problem of propagation of Rayleigh surface waves in a dielectric half-space. Herein we continue along the path of the latter contribution and consider the extension of the Rayleigh–Lamb wave propagation problem to finitely deformed dielectric layers subjected to coupled electromechanical loading.

The work is composed as follows. In Section 2 the theory of finite electroelastodynamics is summarized. The equations for incremental motions superimposed on finite deformations are outlined in Section 3. Specific finite deformations that correspond to three different loading configurations are considered in Section 4 for a layer whose behavior is characterized by a particular augmented energy-density function (AEDF), namely the *incompressible dielectric neo-Hookean* (DH) model. Next, the extension of the Rayleigh–Lamb dispersion relation for a DH layer is introduced in Section 5. An analysis of the dispersion relation is carried out in Section 6, and the effects of the bias electrostatic field and pre-stretch are investigated for the three loading configurations. The main conclusions and observations are summarized in Section 7.

2. Finite electroelasticity

Let $\boldsymbol{\chi}:\Omega_0\times\mathcal{I}\to\Omega\subset\mathbb{R}^3$ describe the motion of a material point $\mathbf{X}$ from a reference configuration of a body Ω_0, with a boundary $\partial\Omega_0$, to its current configuration Ω, with a boundary $\partial\Omega$, by $\mathbf{x}=\boldsymbol{\chi}(\mathbf{X},t)$, where $\mathcal{I}$ is a time interval. The domain of the space surrounding the body is $\mathbb{R}^3\backslash\Omega$ and is assumed to be vacuum. The corresponding velocity and acceleration are denoted by $\mathbf{v}=\boldsymbol{\chi}_{,t}$ and $\mathbf{a}=\boldsymbol{\chi}_{,tt}$, respectively, while the deformation gradient is $\mathbf{F}=\partial\boldsymbol{\chi}/\partial\mathbf{X}=\nabla_{\mathbf{X}}\boldsymbol{\chi}$, and where due to the material impenetrability $J\equiv\det(\mathbf{F})>0$. Vectors between two infinitesimally close points are related through $\mathrm{d}\mathbf{x}=\mathbf{F}\,\mathrm{d}\mathbf{X}$, whereas area elements are transformed via Nanson's formula $\mathbf{N}\,\mathrm{d}A=(1/J)\mathbf{F}^{\mathrm{T}}\mathbf{n}\,\mathrm{d}a$. The volume ratio between an infinitesimal volume element $\mathrm{d}v$ in the deformed configuration, and its counterpart in the reference $\mathrm{d}V$ is given by $\mathrm{d}v=J\,\mathrm{d}V$. As measures of the deformation the right and left Cauchy–Green strain tensors $\mathbf{C}=\mathbf{F}^{\mathrm{T}}\mathbf{F}$ and $\mathbf{b}=\mathbf{F}\mathbf{F}^{\mathrm{T}}$ are used.

Let $\mathbf{e}$ denote the electric field in the current configuration. Commonly the electric field is given by means of a gradient of a scalar field, namely the electrostatic potential. The induced electric displacement field $\mathbf{d}$ is related to the electric field in free space via the vacuum permittivity ε_0 such that $\mathbf{d}=\varepsilon_0\mathbf{e}$. In dielectric media an appropriate constitutive law specifies the relationship between these fields. Generally, this connection can be non-linear and anisotropic.

The balance of linear momentum is

$$\nabla\cdot\boldsymbol{\sigma}=\rho\mathbf{a}, \tag{2.1}$$

where $\boldsymbol{\sigma}$ is the 'total' stress tensor, and ρ is the material mass density. The balance of angular momentum implies that $\boldsymbol{\sigma}$ is symmetric. Note that $\boldsymbol{\sigma}$ consists of both mechanical and electrical contributions, such that the traction $\mathbf{t}$ on a deformed area element can be written as $\boldsymbol{\sigma}\mathbf{n}$ where $\mathbf{n}$ is the unit vector normal to $\partial\Omega$. On the boundary of the material we postulate a separation of the traction into the sum of a mechanical traction $\mathbf{t}_m$ which is a prescribed data, and an electrical traction $\mathbf{t}_e$ which is induced by the external electric field.

Assuming no free body charge (ideal dielectric), Gauss' law reads

$$\nabla\cdot\mathbf{d}=0. \tag{2.2}$$

Under a quasi-electrostatic approximation, appropriate when for the same frequency the length of the waves under consideration are shorter than the electromagnetic waves, Faraday's law states that the electric field is curl-free, i.e.,

$$\nabla\times\mathbf{e}=\mathbf{0}, \tag{2.3}$$

thus enabling the usage of the aforementioned electrostatic potential.

Taking into account fields outside the material, which henceforth will be identified by a star superscript, the following jump conditions should be satisfied across $\partial\Omega$, namely

$$[[\boldsymbol{\sigma}]]\mathbf{n}=\mathbf{t}_m, \tag{2.4a}$$

$$[[\mathbf{d}]]\cdot\mathbf{n}=-w_e, \tag{2.4b}$$

$$[[\mathbf{e}]]\times\mathbf{n}=\mathbf{0}, \tag{2.4c}$$

where w_e is the surface charge density, and the notation $[[\bullet]]=(\bullet)-(\bullet)^\star$ is used for the difference between fields inside and outside of the material. The outer fields are related by

$$\mathbf{d}^\star=\varepsilon_0\mathbf{e}^\star, \tag{2.5}$$

$$\boldsymbol{\sigma}^\star=\varepsilon_0\left[\mathbf{e}^\star\otimes\mathbf{e}^\star-\tfrac{1}{2}(\mathbf{e}^\star\cdot\mathbf{e}^\star)\mathbf{I}\right], \tag{2.6}$$

where $\mathbf{I}$ is the identity tensor. Herein we identify the electrical traction $\mathbf{t}_e$ as the consequence of the external stress $\boldsymbol{\sigma}^\star$, namely the Maxwell stress, such that $\mathbf{t}_e=\boldsymbol{\sigma}^\star\mathbf{n}$. In the surrounding space outside the material $\mathbf{d}^\star$ and $\mathbf{e}^\star$ must satisfy Eqs. (2.2) and (2.3), which reduce to Laplace equation of the electrostatic potential. As a consequence the Maxwell stress is divergence-free.

The foregoing balance and jump equations can be recast in a Lagrangian formulation with the appropriate *pull-back* operations. Specifically, we have that

$$\mathbf{P}=J\boldsymbol{\sigma}\mathbf{F}^{-\mathrm{T}},\quad \mathbf{D}=J\mathbf{F}^{-1}\mathbf{d},\quad \mathbf{E}=\mathbf{F}^{\mathrm{T}}\mathbf{e}, \tag{2.7}$$

for the 'total' first Piola–Kirchhoff stress, Lagrangian electric displacement and electric field, respectively (e.g., [13]). The corresponding balance equations are

$$\nabla_{\mathbf{X}}\cdot\mathbf{P}=\rho_L\mathbf{a},\quad \nabla_{\mathbf{X}}\cdot\mathbf{D}=0,\quad \nabla_{\mathbf{X}}\times\mathbf{E}=\mathbf{0}, \tag{2.8}$$

where $\rho_L=J\rho$ is the density of the material in the reference configuration. The jump conditions across the boundary $\partial\Omega_0$ read

$$[[\mathbf{P}]]\mathbf{N}=\mathbf{t}_M,\quad [[\mathbf{D}]]\cdot\mathbf{N}=-w_E,\quad [[\mathbf{E}]]\times\mathbf{N}=\mathbf{0}, \tag{2.9}$$

where $\mathbf{t}_M\,\mathrm{d}A=\mathbf{t}_m\,\mathrm{d}a$, $w_E\,\mathrm{d}A=w_e\,\mathrm{d}a$ and $\mathbf{N}$ is a unit outward normal to $\partial\Omega_0$.

Following Dorfmann and Ogden [13], the 'total' first Piola–Kirchhoff stress and the Lagrangian electric field are given in terms of an *augmented* energy-density function Ψ (AEDF) with the independent variables $\mathbf{F}$ and $\mathbf{D}$, such that

$$\mathbf{P}=\frac{\partial\Psi}{\partial\mathbf{F}},\quad \mathbf{E}=\frac{\partial\Psi}{\partial\mathbf{D}}. \tag{2.10}$$

For an incompressible material a Lagrange multiplier p is introduced, which is a workless reaction to the kinematic constraint such that

$$\mathbf{P}=\frac{\partial\Psi}{\partial\mathbf{F}}-p\mathbf{F}^{-\mathrm{T}}. \tag{2.11}$$

The latter can be determined only from the equilibrium equations and the boundary conditions.