International Journal of Plasticity xxx (xxxx) xxx–xxx

Contents lists available at ScienceDirect

International Journal of Plasticity

journal homepage: www.elsevier.com/locate/ijplas

Modification of *Mohr*'s criterion in order to consider the effect of the intermediate principal stress

Ana Maria Comanici, Paul Doru Barsanescu*

"Gheorghe Asachi" Technical University of Iasi-Romania, Department of Mechanical Engineering, Mechatronics and Robotics, Blvd. Mangeron, No. 63, 700050, Iasi, Romania

ARTICLE INFO

Keywords:
Yield condition
Metallic material
Analytic functions
Probability and statistics
Virtual *Mohr* circle
Intermediate principal stress

ABSTRACT

Mohr-Coulomb (M-C) criterion is widely used for isotropic brittle materials, although it has some disadvantages which limit its wider application. One important disadvantage of this theory is that it ignores the effect of the intermediate principal stress, although it has an important influence on materials behaviour. Therefore, *Mohr-Coulomb* theory is actually used only for materials in biaxial state of stress. In this paper, a modified form of *Mohr*'s criterion has been suggested, in order to overcome this limitation. The modified criterion takes into account the influence of intermediate principal stress in the case of materials under triaxial loading conditions. On the basis of two new hypotheses, a real triaxial state of stress is replaced by a virtual biaxial state of stress. The above mentioned stress states are considered to be equivalent for a material when the failure is imminent in both cases. Both *Mohr*'s theory and the concept of equivalence of stress states are combined in order to obtain the virtual biaxial state of stress, which is determined using all three principal stresses. The modified *Mohr*'s theory can be used only for triaxial state of stress. For biaxial and uniaxial state of stress it is reduced to the classical one. The proposed hypotheses have an important advantage because does not require additional material coefficients. The calculated values using the virtual biaxial state of stress and the linear M-C criterion are in good agreement with experimental data for grey cast iron. The article also presents a compact statistical analysis for both stresses and errors, in order to objectively assess the effectiveness of the formulated hypotheses.

1. Introduction

The use of new materials, as well as traditional materials, in extreme situations (such as failure behaviour, etc.), requires thorough investigations. For multiaxial state of stress different failure criteria are used. More than 200 failure criteria are known from literature, but a universal failure criterion, able to describe the behaviour of various materials under different complex state of stress, does not exist (Kolupaev, 2018).

For biaxial states of stress, the failure boundary represented in the space σ_1-σ_3 can be a closed curve or a polygon. For ductile materials (without SD effect), the failure limit for tension is approximately equal to the failure limit for compression (in absolute value). Accordingly, the points in which the boundary of the failure surface intersects the axes are symmetrical. Also, the fail boundary is symmetric with respect to the two bisectors, such as, for example, the *Tresca*'s hexagon, *von Mises*' or *Beltrami*'s ellipses etc. (Renani et al., 2016). On the contrary, for materials with SD effect the failure limit for tension is smaller, or even much smaller than the failure limit for compression (in absolute value). Accordingly, the fail boundary of such materials is symmetric only with

* Corresponding author.
E-mail address: paulbarsanescu@yahoo.com (P.D. Barsanescu).

https://doi.org/10.1016/j.ijplas.2018.04.010
Received 25 November 2017; Received in revised form 6 April 2018; Accepted 10 April 2018

respect to the first bisector, which crosses the first and third quadrant, as is the case, for example, of the *Rankine*'s square or the *Mohr-Coulomb* hexagon. The case of materials with SD effect is more general. The *Mohr-Coulomb* criterion, for instance, is reduced to the *Tresca*'s criterion, in the case of materials without SD effect.

Mao-Hong Yu (2002), divides the failure theories or criteria in three categories: those that take into account one shear stress, two shear stresses and respectively three shear stresses. The first category includes *Mohr-Coulomb* (M-C) and Tresca theories; second category is represented by the *Yu* theory, and the third category is described by the *von Mises* theory, etc. (Yu et al., 2006).

Another classification of the criteria can be made according to the number of material parameters used (Altenbach and Zolochevsky, 1996; Kolupaev, 2018). These parameters can be determined by physical experiments. A criterion with a larger number of parameters may have a higher accuracy. However, a large number of tests, required to determine these parameters, represents a disadvantage. Some criteria for isotropic materials, classified according to this criterion, are presented below.

1.1. Six-parameter criteria

H. Altenbach and *A. Zolochevsky* have proposed a generalized failure criterion, based on first invariant of the stress tensor I_1, *von Mises* stress σ_M and six parameters (Altenbach and Zolochevsky, 1996). The limit stress σ_L (or equivalent stress) calculated with this criterion follows with the stress angle ξ:

$$\sigma_L = \lambda_1\sigma_M \sin\xi + \lambda_2\sigma_M \cos\xi + \lambda_3\sigma_M + \lambda_4 I_1 + \lambda_5 I_1 \sin\xi + \lambda_6 I_1 \cos\xi \tag{1}$$

The advantages of *Altenbach-Zolochevsky* criterion are:

- The limit stress is presented explicitly;
- It has a single failure surface in the space of principal stresses, without any plane intersecting.

The important disadvantage of this criterionconsists in the large number of parameters λ_i, which must be determined by the following tests:

- Uniaxial tension;
- Uniaxial compression;
- Pure torsion;
- Thin-walled tubular specimen under inside pressure;
- Biaxial tension;
- Uniaxial tension superposed with hydrostatic pressure.

Of all these tests, the last three are difficult to achieve.

This criterion was applied on grey cast iron, magnesium alloy and some polymers (epoxy).

The simplified forms of this criterion are *von Mises*, *Tresca* and *Capurso* criteria (Kolupaev, 2018).

1.2. Four-parameter criteria

The *Birger*'s criterion (Birger, 1977) is a combination between the model proposed by *Paul* and *von Mises*, and, on the other hand, it is a special case of the generalized criterion proposed by Altenbach and Zolochevsky (1996):

$$\sigma_L = A\sigma_1 + B\sigma_2 + C\sigma_3 + D\sqrt{3J_2} \tag{2}$$

The parameters A, B, C and D are determined by four independent tests: uniaxial tension, uniaxial compression, pure torsion and using tubular specimen loaded by inside pressure. However, the number of tests required and the difficulty of determining the parameter D is a disadvantage of this criterion (Altenbach et al., 2014).

Another criterion that requires four parameters is *Tarasenko* criterion (Altenbach and Zolochevsky, 1996).

1.3. Three-parameter criteria

1.3.1. Paul criterion

B. Paul introduced a modified linear M-C criterion (Altenbach et al., 2014; Meyer and Labuz, 2013; Labuz and Zang, 2012), which takes into account all three principal stresses:

$$\sigma_L = A\sigma_1 + B\sigma_2 + C\sigma_3 \tag{3}$$

This criterion has the disadvantage that it requires three material constants eq. (3) A, B and C, determined by three experiments: uniaxial tension, uniaxial compression, and tubular specimen loaded by inside pressure.

1.3.2. Burzy´nski–Yagn criterion

This criterion has the form: