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On the recoverable and dissipative parts of higher order stresses in strain gradient plasticity

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ABSTRACT

The expressions for the free energy in two recent formulations of strain gradient plasticity are extended to include the locked-in strain energy around statistically stored dislocations. This is accomplished by using the strain dependent factor $\eta(e_p)$, which represents the fraction of the rate of plastic work converted into heat in accordance with the latent heat measurements from classical metal plasticity. The expressions for the plastic work in the two formulations differ by different representations of the portion of plastic work associated with the existence of plastic strain gradients and the corresponding network of geometrically necessary dislocations, while the dissipative parts of plastic work are assumed to be the same in both formulations. The expressions for the recoverable and dissipative parts of the higher order stresses, defined as the work-conjugates to plastic strain and its gradient, are then derived. It is shown that the stress and strain fields of isothermal boundary-value problems of strain gradient plasticity are independent of η, but that this factor may be of importance for non-isothermal analysis in which the dissipated plastic work acts as an internal heat source. The effects of plastic strain gradient on the plastic response of twisted hollow circular tubes made of a rigid-plastic material with different hardening properties are then evaluated and discussed.

1. Introduction

In classical plasticity there is no material length scale in the framework of the constitutive theory, so that this theory cannot predict the size effects experimentally observed in plastic deformation problems at the micron scale, as in the bending and torsion testing of very thin beams and wires, inelastic response of nanograined materials, dispersion strengthening by small particles, measurements of micro-indentation hardness, thin film applications, micro-imprinting processes, etc. (Fleck et al., 1994; Nix and Gao, 1998; Stölken and Evans, 1998; Qiu et al., 2003; Keller et al., 2011; Ma et al., 2012; Liu et al., 2013; Nielsen et al., 2014). In general, the observed trend is that smaller is stronger. This size-dependent strengthening has been attributed to the effects of strain gradients on plastic deformation. The theory which includes these effects has been put forward by Aifantis (1984), Mühlhaus and Aifantis (1991), Fleck and Hutchinson (1993, 1997,2001), and Gao et al. (1999), with subsequent developments by many investigators, including, inter alia, Huang et al. (2000, 2004), Hutchinson (2000, 2012), Gurtin (2002, 2003,2004), Gudmundson (2004), Anand et al. (2005), Gurtin and Anand (2005a,2005b,2009), Bardella (2006, 2007), Fleck and Willis (2009a,2009b), Polizzotto (2009), Voyiadjis et al. (2010), Voyiadjis and Faghihi (2012),

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Dahlberg et al. (2013), Nielsen and Niordson (2014), Mayeur and McDowell (2014), Fleck et al. (2014, 2015), Bardella and Panteghini (2015), and Anand et al. (2015). From the dislocation point of view, the gradient of plastic strain is associated with the storage of geometrically necessary dislocations, while the uniform strain is associated with random trapping and storage of statistically stored dislocations (Ashby, 1970; Fleck et al., 1994; Nix and Gao, 1998; Kysar et al., 2010; Öztop et al., 2013).

In the present paper we extend the strain gradient plasticity analysis of Hutchinson (2012), and Fleck et al. (2014) to include in their expressions for the free energy the locked-in strain energy around statistically stored dislocations. The strain dependent factor $\eta(e_\mathrm{p})$ is used to represent the fraction of the rate of plastic work converted into heat in accordance with the latent heat measurements from classical metal plasticity. The utilized expressions for the plastic work differ by the different representations of the portion of plastic work associated with the existence of plastic strain gradients and the corresponding network of geometrically necessary dislocations, while the dissipative parts of plastic work are assumed to be the same in both formulations. The expressions for the recoverable and dissipative parts of the work-conjugates to plastic strain and its gradient are derived in each case. It is shown that the stress and strain fields of isothermal boundary-value problems of strain gradient plasticity are independent of η, but that this factor may be of importance for non-isothermal analysis in which the dissipated plastic work acts as an internal heat source. The effects of plastic strain gradient on the plastic response of twisted hollow circular tubes made of a rigid-plastic material are then evaluated and discussed. The shear stress, the edge line forces, and the applied torque are determined for various values of the material length parameter. Results for solid rods, hollow and thin-walled tubes are given at the onset and beyond plastic yield for linear and nonlinear hardening.

As in Hutchinson (2012) and Fleck et al. (2014), the presented analysis is phenomenological, without explicit referral to specific dislocation mechanisms and interactions among individual dislocations. The latter are considered in the discrete dislocation dynamics and dislocation based plasticity theory at submicron scales, e.g., Devincre and Kubin (1997), Tadmor et al. (1999), Needleman (2000), Zbib et al. (2002), Bittencourt et al. (2003), Senger et al. (2011), Taheri-Nassaj and Zbib (2015), and Wulfinghoff and Böhlke (2015).

2. Gradient-enhanced effective plastic strain

It is assumed that the elastoplastic rate of strain is the sum of elastic and plastic contributions, such that $\dot\varepsilon_{ij} = \dot\varepsilon^\mathrm{e}_{ij} + \dot\varepsilon^\mathrm{p}_{ij}$. The elastic part of the strain rate depends on the rate of the Cauchy stress (σ_{ij}) according to the generalized Hooke's law. The plastic part of the strain rate is assumed to be codirectional with the deviatoric part of stress (σ'_{ij}), as in the classical J_2 flow theory of plasticity,

$$\dot\varepsilon^\mathrm{p}_{ij} = \dot e_\mathrm{p} m_{ij}\,, \quad m_{ij} = \frac{3}{2}\frac{\sigma'_{ij}}{\sigma_\mathrm{eq}}. \tag{1}$$

The equivalent stress is $\sigma_\mathrm{eq} = [(3/2)\sigma'_{ij}\sigma'_{ij}]^{1/2}$, while the loading index satisfies

$$\dot e_\mathrm{p} = \left(\frac{2}{3}\dot\varepsilon^\mathrm{p}_{ij}\dot\varepsilon^\mathrm{p}_{ij}\right)^{1/2}. \tag{2}$$

Its path-dependent integral over the history of deformation gives the effective plastic strain e_p. The spatial gradient of e_p will be used as a cumulative measure of plastic strain gradients, so that

$$e_\mathrm{p} = \int_0^t \dot e_\mathrm{p}\,\mathrm{d}t\,, \quad e_{\mathrm{p},k} = \int_0^t \dot e_{\mathrm{p},k}\,\mathrm{d}t. \tag{3}$$

In the strain gradient plasticity, a gradient-enhanced effective plastic strain can be defined by Hutchinson (2012)

$$E_\mathrm{p} = \left(e_\mathrm{p}^2 + l^2 e_{\mathrm{p},k}e_{\mathrm{p},k}\right)^{1/2}, \tag{4}$$

where l is the material length scale of the specific problem at hand, introduced in (4) on the dimensional ground. While e_p is a monotonically increasing measure of plastic strain during the course of plastic deformation, $e_{\mathrm{p},k}$ is not necessarily increasing because $\dot e_{\mathrm{p},k}$ can be negative for certain non-proportional strainings, so that the gradient-enhanced plastic strain E_p is not necessarily an increasing measure of strain either (i.e., $\dot E_\mathrm{p}$ could be negative).

In the classical J_2 flow theory of plasticity, the rate of plastic work (per unit volume) is $\dot w_\mathrm{p} = \sigma'_{ij}\dot\varepsilon^\mathrm{p}_{ij} \equiv \sigma_0(e_\mathrm{p})\dot e_\mathrm{p}$, where $\sigma_0 = \sigma_0(e_\mathrm{p})$ is the stress-plastic strain curve in uniaxial simple tension test, and $\sigma_\mathrm{eq}=\sigma_0(e_\mathrm{p})$ is the yield condition. In the strain gradient plasticity it has been proposed (Hutchinson, 2012) that the specific plastic work is