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Study on the definition of equivalent plastic strain under non-associated flow rule for finite element formulation

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ABSTRACT

As opposed to associated flow rule (AFR) in which yield function and plastic potential are equal, the different definitions for them is an inherent characteristic of non-associated flow rule (non-AFR). This imposes a specific relation (but not equality) between equivalent plastic strain and plastic compliance factor. Unavoidably, this leads to a laborious effort for FE implementation of non-associated constitutive model specifically when several internal variables (such as kinematic hardening or damage parameters) are involved. This paper is mainly devoted to studying the conditions at which the non-AFR approach can be simplified so that the numerical implementation scheme is more convenient without loss of accuracy. It will be shown that by scaling the plastic potential function, the equality of equivalent plastic strain and compliance factor can be reserved. The effect of scaling of the non-AFR based on Barlat et al.'s (2003) anisotropic model (called Yld2000-2d) is comprehensively studied with FE simulation of tensile loading under uniaxial tensions along the different orientations as well as balanced biaxial stress condition. A fully implicit return-mapping scheme was introduced for stress integration of the constitutive model in a User-defined MATerial subroutine (UMAT). Cup drawing simulations of a highly textured aluminum alloy 2090-T3 were performed using simplified and original approaches. The results prove that the proposed simplified technique is a reliable alternative for the full expression.

1. Introduction

Various phenomenological yield functions have been proposed to simulate the anisotropic behaviors of metals. Most anisotropic yield functions are based on the associated flow rule (AFR) hypothesis obeying the normality rule. Accordingly, under the assumption of AFR with the light of material orthotropy, various phenomenological yield functions have been proposed to describe the initial anisotropy of metallic sheets including Hill's (1948), Barlat et al. (1991, 2003, 2005, 2007), Karafillis and Boyce (1993), Cazacu and Barlat (2002, 2004), Bron and Besson (2004), Cazacu et al. (2004, 2006), Vegter and van den Boogaard (2006), Hu (2007), etc. Barlat et al. (2011) proposed an alternative approach to consider kinematic hardening within the framework of anisotropic yield function under associated flow rule. Recently, Cleja-Tigoiu and Iancu (2013) considered an orthotropic non-quadratic yield function dependent on the third invariant of the stress and showed that the plastic spin provides the change in the orthotropy axes, characterized by the Euler angles.

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During the last decade, some attention has been paid on the development and implementation of non-AFR for metal plasticity. Stoughton (2002) proposed a non-AFR model based on Hill (1948) quadratic formulation that accurately predicted both direction dependent *r*-values and yield stresses in rolling, transverse and diagonal directions. Continuing his previous model, Stoughton and Yoon (2004) developed a pressure sensitive non-AFR model that predicted the strength differential effect observed in tension and compression tests. Stoughton and Yoon (2006, 2008) derived the stability conditions for non-associated flow plasticity. Cvitanic et al. (2008) developed a non-AFR model based on both Hill (1948) and Karafillis and Boyce non-quadratic yield functions combined with isotropic hardening, which demonstrated improved height predictions for deep drawn cups. Stoughton and Yoon (2009) proposed a non-AFR based anisotropic hardening model that resulted in excellent predictions of hardening curves for rolling, transverse and diagonal directions and for the balanced biaxial stress state. Improvements in prediction of cup height and springback of a U-bend specimen using non-AFR with mixed isotropic-kinematic hardening have been also reported by Taherizadeh et al. (2010, 2011). Gao et al. (2011) showed the significance of the hydrostatic stress on plastic response with the non-associated flow rule. Park and Chung (2012) derived a symmetric stiffness modulus for the non-associated flow rule under the framework of the combined isotropic-kinematic hardening law. Recently, Safaei et al. (2013a) proposed an evolutionary anisotropic model based on non-AFR that excellently predicted distortional hardening and evolution of instantaneous *r*-values in seven uniaxial directions as well as balanced biaxial loading condition.

Among many benefits of using non-associated flow for anisotropic plasticity, non-AFR hypothesis removes the constraint of the normality rule where plastic potential and yield function are equal under associated flow rule. Consequently, two separate functions for yield and plastic potential can be adopted. In other words, the yield function and plastic potential describe the elastic limit and plastic strain rate direction independently. Then, the hardening of yield function and the direction of plastic flow can be separated resulting in the uncoupled predictions of stress ratios and *r*-values.

Considering the increasing popularity of non-AFR in metal forming simulations, this paper is aimed to propose a simple implementation based on fully implicit integration scheme. The motivation is that the combination of a non-AFR model with complex hardening model or damage parameters is a cumbersome task due to inequality of plastic compliance factor and equivalent plastic strain. Consequently, this paper proposes a method to reduce the degree of inaccuracy with simplification. First, we briefly discuss the development of a non-AFR Yld2000-2d (Barlat et al., 2003). Then, the stress and Lankford directionalities predicted from AFR and non-AFR Yld2000-2d are compared for two highly anisotropic materials such as an interstitial free steel DC06 and AA2090-T3. The developed (simplified and full) models were implemented into a commercial FE code ABAQUS using a fully implicit return mapping algorithm (backward Euler method). The multi-stage return mapping method based on the incremental deformation theory proposed by Yoon et al. (1999) is used in the implementation to enhance the convergence of the linearization algorithm for large strain increments. Subsequently, comprehensive comparisons are provided for the simplified and full methods. Finally, the results of cup deep drawing simulations based on the evaluated models are discussed.

2. Review of associated flow rule (AFR)

Associated flow rule (AFR) reflects the normality rule that describes that the gradient of the yield surface determines the direction of plastic flow. We can write a yield criterion under AFR as

$$F = f_y(\underset{\sim}{\sigma}) - \sigma^{iso}(\bar{\varepsilon}^p) \tag{1}$$

where f_y and σ^{iso} respectively denote yield function and isotropic hardening; $\bar{\varepsilon}^p$ is equivalent plastic strain. The normality rule is given by

$$d\underset{\sim}{\varepsilon}^p = d\lambda\, \underset{\sim}{m} \tag{2}$$

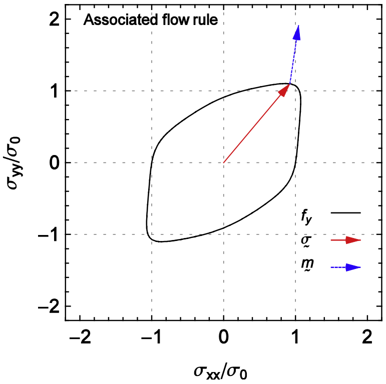

Fig. 1. Concept of normality rule (f_y is yield function; $\underset{\sim}{\sigma}$ is Cauchy stress; $\underset{\sim}{m}$ is plastic strain rate direction).