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Strength and buckling of an untypical dished head of a cylindrical pressure vessel

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ABSTRACT

The paper is devoted to an untypical dished head equivalent to a thin-walled shell of revolution. The goal of analysis is the elimination of the edge effect arising at the joint of the cylindrical shell and the dished head. The meridian of the head is a composite curve of a circular arc and a polynomial of the fifth degree. The stress state in the dished head is analytically and numerically-FEM studied. Moreover, the buckling problem of the head is numerically calculated.

1. Introduction

The shape of the dished head of pressure vessels has a significant influence on the stress state and critical loads. This problem has been studied and described in many works. Authors of the monograph [1] presented the concepts and principles for design of pressure vessels. Ventsel and Krauthammer [2] collected and described in details the strength, buckling and vibration problems of thin-walled plates and shells. Calladine [3] provide a complex knowledge concerning geometry of shells as well as stress and stability problems. Findlay and Timmins [4] presented a parametric study of the stress state and design of tori-conical heads. Sorić [5] studied imperfection sensitivity to strength of internally pressurized tori-spherical shells. Magnucki and Lewiński [6] formulated the problem of shaping of dished head considering the membrane stress state. Magnucki et al. [7] analytically calculated the minimum of stress concentration factor in cylindrical pressure vessels with ellipsoidal heads. Malinowski and Magnucki [8] presented the optimal design of a sandwich ribbed flat baffle plate of a circular cylindrical tank. Krivoshapko [9] described the elastic buckling problems of ellipsoidal shells with regard to pressure vessels. Mackenzie et al. [10] presented the elastic-plastic deformation and buckling of tori-spherical pressure vessel heads under static pressure. Błachut and Magnucki [11] presented the review of selected problems relating to the strength, buckling and optimization of pressure vessels. Carbonari et al. [12] described a design of pressure vessels using shape optimization with multi-objective function. Krużelecki and Proszowski [13,14] studied the optimal shaping of dished heads of pressure vessels under internal pressure. Banichuk [15] described the approach to optimize the meridional shape and the thickness variation for shells of revolution. Błachut [16] presented the elastic buckling of steel domes with initial shape imperfections under external pressure. Jasion and Magnucki [17] theoretically studied an elastic buckling problem of clothoidal-spherical shells under external pressure. Another example of an untypical shape of shell is the egg-shaped shell analysed by Zhang et al. [18]. Authors present the buckling and post-buckling behaviour of a family of such shells. Zingoni [19,20] and Zingoni et al. [21,22] presented an analytical approach to analyses of new shapes of shell of revolution. The problem of bending stresses as well as the edge effect was taken into account.

The subject of the present paper is the thin-walled untypical dished head of a cylindrical pressure vessel. The meridian of this head is a composite curve of a circular arc and a polynomial of the fifth degree. Thus the first part of the head is a spherical shell, while the second part is a special shell of revolution (Fig. 1).

The typical hemispherical, tori-spherical or ellipsoidal heads disturb the membrane stress state in the vessel – the edge effect occurs in the region of the joint of these heads with the cylindrical shell. The goal of the following investigation is to optimize the shape of the second part of the head, the curve *M-N*, to minimize this edge effect. The preliminary results of the work have been published in conference proceedings Magnucki et al. [23].

2. Analytical shaping of the untypical dished head of the cylindrical vessel

The initial part of the meridian is a circular arc of radius R_0 and angle β_0 (Fig. 1), whereas the part between the points M and N is a curve described by the polynomial of fifth degree which allows to fulfil

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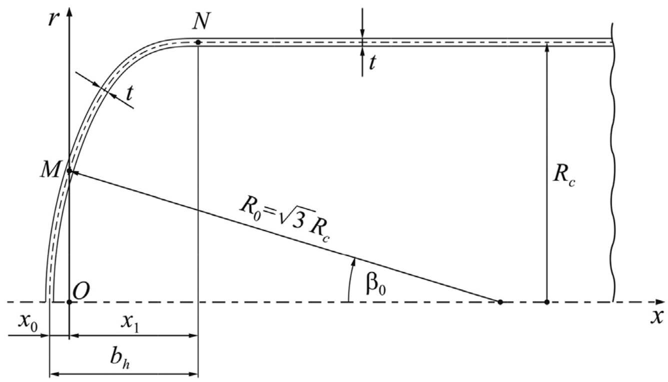

Fig. 1. Scheme of the untypical dished head of the cylindrical vessel.

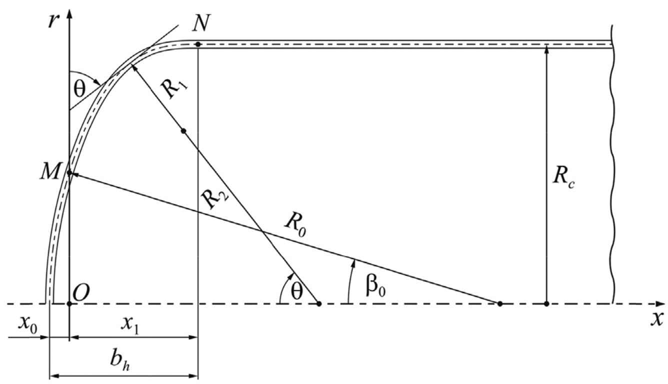

Fig. 2. Scheme of the principal radii R_1 and R_2 of curvatures and the angle coordinate.θ

Table 1
The selected values of the dimensionless meridional stresses (14).$\tilde{\sigma}_\theta^{(M-N)}$

ξ	0	0.1965	0.4814	0.6457	0.6651	0.8749	0.9153	1
$\tilde{\sigma}_\theta^{(M-N)}$	$\sqrt{3}/2$	0.7410	0.5723	0.5245	0.5206	0.5005	0.5000	0.5

Table 3
The selected values of the dimensionless equivalent stresses (17).$\tilde{\sigma}_{eq}^{(M-N)}$

ξ	0	0.1965	0.4814	0.6457	0.6651	0.8749	0.9153	1
$\tilde{\sigma}_{eq}^{(M-N)}$	$\sqrt{3}/2$	0.7410	0.8270	$\sqrt{3}/2$	0.8644	0.5005	0.4330	$\sqrt{3}/2$

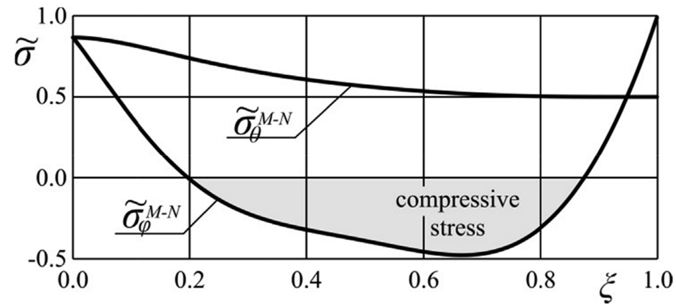

Fig. 3. The meridional $\tilde{\sigma}_\theta$ and circumferential $\tilde{\sigma}_\phi$ dimensionless stresses in the head.

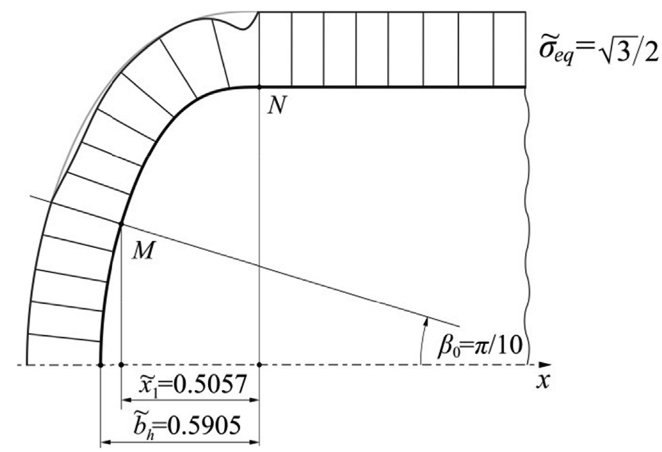

Fig. 4. The equivalent Huber-Mises dimensionless stresses $\tilde{\sigma}_{eq}$ in the head.

the consistency conditions listed below. The form of the polynomial is as follows

$$r(x) = R_c \sum_{j=0}^{5} \alpha_i \xi^j, \tag{1}$$

where: $\xi = x/x_1$ – dimensionless coordinate $(0 \le \xi \le 1)$, α_i – dimensionless coefficients.R_c – radius of the cylindrical shell.

The depth of the dished head

$$b_h = x_0 + x_1 \tag{2}$$

where: $x_0 = \sqrt{3}(1 - \cos\beta_0)R_c$ – depth of the circular arc, x_1 – depth of the curve ***M-N***.

Thus, the relative depth of the dished head

$$\tilde{b}_h = \frac{b_h}{R_c} = \tilde{x}_0 + \tilde{x}_1 \tag{3}$$

where $\tilde{x}_0 = \sqrt{3}(1 - \cos\beta_0)$, $\tilde{x}_1 = x_1/R_c$.

The consistency conditions at the joint of the curve (1) with the circular arc in the point *M* and with the straight line in the point *N* are as follows:

$$\xi = 0), \quad r(0) = \alpha_0 R_c = \sqrt{3} R_c \sin\beta_0, \tag{4}$$

$$\xi = 0), \quad \left.\frac{dx}{dr}\right|_0 = \frac{x_1}{\alpha_1 R_c} = \tan\beta_0, \tag{5}$$

$$\xi = 1\Big), \quad r(1) = R_c \sum_{j=0}^{5} \alpha_j = R_c, \tag{6}$$

$$\xi = 1), \quad \left.\frac{dr}{dx}\right|_1 = R_c \sum_{j=1}^{5} j\alpha_j = 0. \tag{7}$$

Moreover, the consistency conditions of the principal radii of curvatures for the joints of three surfaces of revolution (Fig. 2) are as follows:

$$\xi = 0), \quad R_1(0) = R_2(0) = \sqrt{3} R_c, \tag{8}$$

$$\xi = 1), \quad R_1(1) \to \infty, \quad R_2(1) = R_c, \tag{9}$$

where the principal radius of the meridian – the curve (1)

Table 2
The selected values of the dimensionless circumferential stresses (14).$\tilde{\sigma}_\phi^{(M-N)}$

ξ	0	0.1965	0.4814	0.6457	0.6651	0.8749	0.9153	1
$\tilde{\sigma}_\phi^{(M-N)}$	$\sqrt{3}/2$	0	−0.3759	−0.4749	−0.4772	0	0.2502	1.0