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Calculation of stress intensity factors for circumferential semi-elliptical cracks with high aspect ratio in pipes

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ABSTRACT

In this paper, stress intensity factors are calculated at the deepest point of an internal circumferential semi-elliptical crack in a pipe subjected to any arbitrary load. Based on the three dimensional finite element analysis, a weight function is proposed for high aspect ratio semi-elliptical cracks in pipes. An effective expression is developed analytically to evaluate the stress intensity factor using the weight function method. For several crack face stress fields and welding residual stress distributions, the weight function is validated against finite element data and those in the literature. Based on the comparison results, it can be concluded that the solution proposed in this paper is effective in engineering applications.

1. Introduction

Cylindrical vessels are widely used in industries. Most of the time, cracks can be observed in this type of structures. In addition to manufacturing faults, pitting corrosion can induce crack within the vessels. During crack growth, initially irregular crack front will rapidly become approximately semi-elliptical, so the geometrical shape of a surface flaw or pit can conform to elliptical geometry. As a result, analysis of surface cracks in pipes represents one of the most important problems in the integrity evaluation of flawed structural components. Because of pitting corrosion process, stress intensity factor solutions for internal circumferential surface cracks in pipes are of great technical importance from an industrial point of view (Fig. 1). However, to the authors' knowledge, there are still no exact solutions for the stress intensity factors for surface cracks up to today.

There are relatively few solutions for pipes containing semi-elliptical circumferential cracks. Most of the research conducted has used finite element analysis to determine the stress intensity factors. Kumar et al. [1] obtained the elastic analysis of part-through axial and circumferential cracks in cylinders using the line-spring model and shell finite element method. Xian-Ming et al. [2] presented an empirical equation for the stress intensity factors for circumferential surface cracks in a cylinder under remote tension based on the results obtained from photoelastic experiments and caustic method. Bergman and Brickstad [3] calculated the stress intensity factors for a circumferential surface crack using the finite element analysis and line-spring method. The crack was modelled with a constant depth and not semi-elliptical. Poette and Albaladejo [4] determined the stress intensity factors for semi-elliptic circumferential cracks in pipes using virtual extension method. Bergman [5] employed the finite element method to calculate the stress intensity factors for a range of crack sizes and six different load cases. The derived solutions cover a wide range of geometry and load configurations. Wallbrink et al. [6] used a semi-analytical solution technique for predicting the stress intensity factors around a partly circumferential elliptical surface crack in a pipe. The technique involved the use of a conformal transform in conjunction with a finite element model to determine stress intensity factors. El Hakimi et al. [7] solved the surface crack problem in a cylinder and a sphere using the finite element method. The J integral was employed to consider both elastic and elastoplastic behaviors.

Kamaya and Nishioka [8] developed an analysis system using the finite element alternating method for evaluating the stress intensity factors of circumferential and longitudinal surface cracks in a cylinder. Zahoor [9] presented closed form stress intensity factor expressions for five flaw cases. The solutions were taken from finite element data and curve fitting. Oh et al. [10] estimated stress intensity factors using the finite element solutions for two types of generic welding residual stress profiles, generated by simulating similar and dissimilar metal welds with repair welds. A third-order

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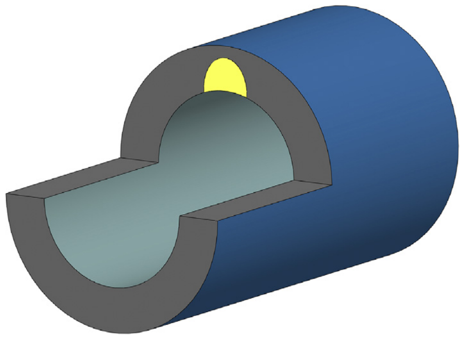

Fig. 1. Schematic view of an internal deep circumferential semi-elliptical crack in a pipe.

fitted polynomial equation is used with influence coefficients for calculating the stress intensity factor in Appendix C of API 579 Standard Code [11]. Miyazaki and Mochizuki [12] computed the stress intensity factors of surface cracks, in plate and pipes under two assumed residual stress fields by the simplified equation in API. Shahani and Nabavi [13–15] have solved the mechanical and thermal stress intensity factors of an axial semi-elliptical crack using the weight function method.

The surveyed literature revealed the fact that there is almost no complete solution for the stress intensity factors for semi-elliptic circumferential cracks with the aspect ratio higher than unity located at the inner wall of a cylinder. In this paper, closed-form stress intensity factors are derived for a deep circumferential semi-elliptical crack in a cylinder with the aid of the weight function method. First a three-dimensional finite element modeling is employed utilizing displacement-based quarter-point singular elements to calculate the stress intensity factors for different geometries. Because of rapid changes in the geometrical parameters around the crack front region, fine mesh should be used for modeling this region. Then, using the results of the finite element analysis as reference loads, the weight function for the deepest point of the circumferential semi-elliptical crack with high aspect ratio (a/c > 1) is derived. The benefit of the proposed weight function is the ability of the calculation of closed-form stress intensity factors for a desired continuous loading. The nth-order representation of stress distribution will allow without any limitations more accurate fitting of highly non-linear stress distributions, such as those depicting weld residual stress fields involving multiple peaks and valleys. An effective closed form expression in conjunction with the nth-order polynomial stress distribution is proposed to evaluate the stress intensity factors for semi-elliptic circumferential cracks with high aspect ratios in pipes. Comparing the results with those published in the literature in special cases, shows the efficiency of the proposed method and reveals the possibility of determining the stress intensity factor by the given closed form expression.

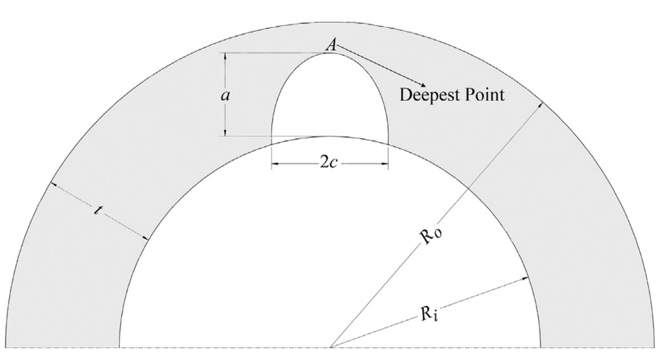

Fig. 2. Definition of parameters for an internal circumferential semi-elliptical crack with high aspect ratio in a pipe.

2. Finite element modeling

The three-dimensional analysis is performed on a pipe of inner radius R_i, outer radius R_o, and wall thickness t. The cylinder contains a deep circumferential semi-elliptical crack of length $2c$ and depth a (see Fig. 2). Due to the symmetry, only a quarter of the pipe was analyzed by the finite element method. The finite element program ABAQUS [16] was used for the calculations. The square-root singularity of stresses is modeled by shifting the mid-point nodes to the quarter-point locations in the region around the crack front. This area is meshed with collapsed isoparametric elements and the rest of the model is covered with 20-node elements. As shown typically in Fig. 3 the size of elements should be small enough near the crack front to ensure the accuracy of the results. For each crack geometry i.e., the aspect ratio (a/c) and the relative depth (a/t), the stress intensity factors may be calculated from the ABAQUS built in J-integral solver based on the domain integral technique. Having performed convergence tests, reliability of each finite element model of the cracked cylinder is obtained by path independence checking of J values. Also, independency of the obtained stress intensity factors to the elements number is checked.

The plane strain relationship between K and J is used to calculate the stress intensity factor as follows

$$K_I = \sqrt{\frac{EJ}{1-\nu^2}} \tag{1}$$

The aspect ratio of the semi-elliptical crack changes from 1 to 2 and the relative depth varies between 0.2 and 0.8. The ratio of the external to the internal radii of the cylinder in all examples is assumed to be $R_o/R_i = 1.10$. The loads were applied to crack surface with the following stress distribution

$$\sigma_{ref,m} = \sigma_0\,(x/a)^m \quad m = 0,\ 1 \tag{2}$$

where σ_0 is the nominal stress and a is the crack depth. In each case the $x = 0$ corresponds to inner surface of the pipe and $x = a$ corresponds to the deepest point of the crack. The boundary correction factors can be expressed as follows

$$Y_m = K_{ref,m}\Big/\left(\sigma_0\sqrt{\pi a/Q}\right) \tag{3}$$

where $K_{ref,m}$ is the stress intensity factors for each given loading case and Q is the shape factor for a high aspect ratio elliptical crack as follows

$$Q = 1 + 1.464\,(c/a)^{1.65} \tag{4}$$

Expressions for the Y_0 and Y_1 are obtained from the calculated finite element data for the *deepest* point of the circumferential semi-elliptical crack. The unique curve-fitting to appropriate functions for each case of the crack geometric function for both uniform and linear loading can be extracted as

$$Y_m = \sum_{i=1}^{6}\sum_{j=1}^{5} A_{mij}\,(a/c)^{i-1}(a/t)^{j-1} \quad , \quad m = 0,\ 1 \tag{5}$$

where the parameters A_{mij} are shown in Table 1 for uniform loading $(m = 0)$ and linear loading $(m = 1)$. The above expressions were fitted to the finite element data with an average accuracy better than 1% for aspect ratio $1.0 \le a/c \le 2.0$.