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# Stress intensity interaction between dissimilar semi-elliptical surface cracks

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ABSTRACT

Procedures for structural integrity assessment normally contain criteria to predict the significance of the interaction between neighbouring defects in a structure. Here, the elastic interaction between coplanar semi-elliptical surface cracks is examined in detail by considering a large number of dissimilar crack pairs with different depths and aspect ratios. Surface defect interaction criteria from several assessment procedures are critically assessed and found to be satisfactory for cracks loaded in uniform tension. The criterion used in the R6 Rev. 4 and BS 7910:2013 procedures is the least inherently conservative of those considered here. However, the amount by which interaction exacerbates the most severe crack front loading state can depend strongly on the distribution of stress applied to the cracks. This means that the loading mode should be taken into consideration when judging whether the interaction between surface defects is significant.



## 1. Introduction

In structural integrity analysis it is often necessary to predict the combined effect of two or more flaws in a structure. As a result, integrity assessment procedures such as the British assessment standard BS 7910:2013 [1], the R6 Rev. 4 procedure maintained by EDF Energy and others [2], and the ASME Boiler and Pressure Vessel Code Section XI [3] contain rules for analysing adjacent defects, in addition to guidance on how to predict their combined effect on a structure. These procedures are designed to cover various failure mechanisms including brittle fracture, elastic-plastic fracture and plastic collapse. However for simplicity and conservatism, rules for considering the interaction between adjacent defects are normally based on linear elastic fracture mechanics analyses. In practice this means determining how the stress intensity factor which occurs at one crack is influenced by the presence of an additional crack or defect close by.

The problem of interacting co-planar semi-elliptical surface cracks is a particularly important one because defects due to stress-corrosion cracking, fatigue, and weld cracking can often be approximated using this geometry. For co-planar surface defects, assessment codes typically provide rules for conservatively characterising the defects as semi-elliptical or rectangular cracks. Interaction criteria based solely on the defect geometry have been established using the results of numerical stress analyses in conjunction with the relatively scarce experimental data which exists for these cases [4–6].

The analysis of interacting cracks in a linear elastic material has developed steadily in response to improvements in capability for computational stress analysis. The most important early work on this problem used a 'body force' method of analysis based on equivalent Eshelby-type ellipsoidal inclusions [7,8]. This method is computationally efficient and allows analysis of a wide range of different crack sizes and aspect ratios, but it is best suited to analysis of cracks emanating from the free surface of an infinite half-space rather than cracks in plates and shells of finite thickness. Additionally, for materials with Poisson's ratios in a practical range (i.e. $\nu \approx 0.3$ for metals) it is difficult to derive accurate stress intensity factor results for points on the crack front close to an intersection with a free surface using the body force method [9]. The line-spring analysis developed by Rice and Levy [10] can be coupled with the boundary element method to yield results for semi-elliptical cracks. Zeng et al. [11] used this technique to analyse pairs of identical surface cracks, presenting a comparison between this and the crack pair re-classified as a single crack by the method given in ASME BVPC Section XI [3].

For cracks in finite-thickness plates with realistic elastic properties, the finite element method has proven to be a versatile technique despite entailing a greater computational cost than the

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body force and line-spring methods. Finite element results for interacting semi-elliptical cracks are presented by Soboyejo et al. [12], Stonesifer et al. [13] and Yoshimura et al. [14]; with the latter study in particular showing that FEA can be used to rapidly generate results for a large number of crack geometries. More recent finite element studies including those by Sethuraman et al. [15,16], and Carpinteri et al. [17] have used this method to determine stress intensity factors and interaction parameters for a range of semi-elliptical crack geometries. From this work, it is evident that the magnitude of the interaction between two cracks depends strongly on the distance between them and quickly becomes insignificant as this distance is increased [18]. Use of the finite element method also makes it practical to model crack growth and coalescence for processes such as fatigue which are driven by the stress field at the crack tip. Results from models of this type, modelling crack propagation from multiple initially semi-elliptical defects, have been presented in studies by Kishimoto et al. [19] and Lin & Smith [20].

So far, the majority of work on coplanar surface cracks has concentrated on studying the interaction between two identical defects. This greatly simplifies the problem of determining whether interaction between the cracks is significant enough to be considered in subsequent analysis. For a pair of dissimilar defects, there are far more possible combinations of crack depth and aspect ratio to be considered. Likewise, there has been a focus on the simplest defect loading modes: uniform tension and bending. In reality, non-linear variations in stress through the thickness of plates and shells frequently occur, often as a result of residual and thermal stresses. Although some researchers, such as Carpinteri et al. [17] have investigated the effects of these non-uniform loadings, interaction criteria typically do not include any dependence on loading mode.

The purpose of this study is to evaluate the criteria that are used within structural integrity assessment procedures to judge the significance of crack interaction effects. Using the results of finite element models of a broad range of dissimilar crack pairs, these criteria can be examined more thoroughly than when only data for pairs of identical defects is available. The effects of the through-thickness distribution of stress and the material's elastic properties on the effectiveness of interaction criteria have also been identified.

## 2. Stress intensity factor determination

### 2.1. Notation and conventions

Fig. 1 shows the basic geometry of a pair of dissimilar semi-elliptical defects emanating from the same surface on a plate of unit thickness. For convenience, the cracks are numbered 1 and 2. Crack 1 is always the deeper of the two if they have different depths (ie. $a_1 \geq a_2$) and is located positive in $x$ relative to Crack 2. It is also convenient to parameterise the geometry of the crack pair using the following normalised factors: $\xi = a_1/b$ is the non-dimensional depth of Crack 1, $\beta = a_2/a_1$ is the depth of Crack 2 relative to Crack 1, $\alpha_1 = a_1/c_1$ is the aspect ratio of Crack 1, $\alpha_2 = a_2/c_2$ is the aspect ratio of Crack 2, and $\delta = d/b$ is the non-dimensional distance between the two cracks. For any pair of semi-elliptical cracks on the same side of a plate, $\xi < 1$, $\beta \leq 1$, $\alpha_1 > 0$, $\alpha_2 > 0$ and $\delta > 0$.

A point on either semi-ellipse can be defined using its parametric angle $\phi$, as shown in Fig. 2. For each crack, $\phi$ is measured from the intersection point of the crack front with the plate surface closest to the other crack. This means that for cracks on the same side of the plate, Crack 1 has $\phi_1$ measured anticlockwise-positive whereas for Crack 2, $\phi_2$ is measured clockwise-positive.

To examine the interaction of the two cracks, an interaction factor $\gamma$ can be defined as [17]:

$$\gamma(\phi) = \frac{K_I^{int}(\phi)}{K_I^{\delta=\infty}(\phi)} \tag{1}$$

where $K_I^{int}$ is the Mode I stress intensity factor at the interacting crack and $K_I^{\delta=\infty}$ is the Mode I stress intensity factor for a crack of the same geometry and under the same loading conditions, but remote from any other defect. Since the Mode I stress intensity factor for each crack varies as a function of position over the crack front, $\gamma$ is a function of $\phi$. Examples showing the variation in $\gamma(\phi)$ across the crack front in pairs of identical cracks are given in Section 3.1 (Figs. 4 and 5). However, in general Crack 1 and Crack 2 may have differing depths and aspect ratios, and consequently they may have differing interaction factor functions. These can be written as:

$$\gamma^N(\phi) = \frac{K_I^{N,int}(\phi)}{K_I^{N,\delta=\infty}(\phi)} \tag{2}$$

where the superscript $N$ may take the value 1 or 2 to indicate values for Crack 1 and Crack 2 respectively. For example $K_I^{2,\ int}$ denotes the Mode I stress intensity factor on Crack 2 when it is in proximity to the other crack. For assessing whether or not the interaction between two cracks is significant it is useful to further define a 'global' interaction factor $\gamma^G$. This is the factor by which the maximum Mode I stress intensity factor present anywhere on either crack line is increased by proximity of the cracks to one another:

$$\gamma^G = \frac{\max\limits_{\phi,N} K_I^{N,\ int}(\phi)}{\max\limits_{\phi,\ N} K_I^{N,\ \delta=\infty}(\phi)} \tag{3}$$

This quantity represents the amount by which the most unfavourable condition on the crack pair (according to single-parameter linear elastic fracture mechanics) has been exacerbated by interaction between the cracks.

### 2.2. Finite element analysis

The Abaqus/CAE finite element pre-processor [21] working in conjunction with custom code written in MATLAB [22] and Python was used to automatically generate individual finite element models for a large number of different crack pairs. For each crack pair, model geometry information including the crack positions, mesh transition positions, element sizes etc. was defined using the basic parameters $\xi$, $\beta$, $\alpha_1$, $\alpha_2$ and $\delta$ (defined in Section 2.1). This geometric information was written into a Python script specifying the process required to generate a model via the Abaqus/CAE scripting interface. The script was executed, causing Abaqus/CAE to create and mesh a model, and write an input file which could be passed to the FE solver. All of the analyses were performed using the Abaqus/Standard 6.12 solver [23] on a server machine with 12 Intel Xeon ×5670 CPUs and 50 GB of RAM running under CentOS Linux 6.8. Further MATLAB code was used to control the execution of models and extract results from the output files.

Since the solid body is symmetric about the plane containing the cracks, it is only necessary to model one half of it. The nominally infinite plate containing the cracks was approximated using a finite plate which was large in comparison to the region containing the cracks: 1000 × 1000 units in breadth and half-length for a plate of unit thickness. Three types of mesh generation region were used. The crack tip region (Region 1) consists of 8-noded reduced integration linear brick elements arranged in a layer five elements thick, which surround a set of 6-noded linear wedge elements at the crack tip. 50 elements were used along the length of each semi-