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Global limit load solutions for plates with surface cracks under combined biaxial forces and cross-thickness bending

Yuebao Lei*, Peter Budden

EDF Energy Nuclear Generation Ltd., Barnett Way, Barnwood, Gloucester GL4 3RS, UK

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ABSTRACT

Lower bound limit load solutions for surface cracks in plates under combined end force, cross-thickness bending moment and tensile/compressive membrane stress parallel to the crack are derived based on the von Mises yield criterion. From these solutions, particular limit loads for plates with extended surface cracks and through-thickness cracks or uncracked plates under the same loading conditions are obtained. The limit load solutions for surface cracks in plates under combined tension and bending due to Lei and Fox can be reproduced from the solutions in this paper by setting the stress parallel to the crack plane to zero.

1. Introduction

In an R6 [1] structural integrity assessment, both fracture and plastic collapse of components containing crack-like flaws are assessed using the failure assessment diagram (FAD) method. For plastic collapse assessment, the applied load is directly compared with the flow stress based limit load of the defective structure. The fracture assessment using the FAD method is underpinned by the reference stress *J*-estimation scheme [2], where the reference stress is defined as the yield stress of the material multiplied by the ratio of applied primary load to the limit load of the defective component. Therefore, the limit load is one of the most important inputs when a structural integrity assessment is performed using R6 [1] type procedures as it gives the load-carrying capacity for plastic collapse of the defective component and also defines the *J*-integral via the reference stress method for elastic–plastic fracture. Limit load solutions for various defective geometries and various load combinations have been continuously developed under the R6 development programme. This paper addresses the effect of stress parallel to the crack plane on the limit load and hence on *J* predictions via the reference stress method.

In a structural integrity assessment, a surface defect in a structure may often be conservatively simplified to a surface crack in a plate subjected to combined loading. In the current version of R6 [1], global limit load solutions for surface cracks in plates under combined tension and positive bending, i.e. the applied moment tends to open the crack, are recommended, based on Goodall and Webster [3] and Lei [4]. Recently, Lei and Fox [5] extended this solution to any loading combination of tensile/compressive end force and positive/negative cross-thickness bending. However, the effect of stresses parallel to the crack plane on plastic collapse has not yet been addressed.

Miura and Takahashi [6] proposed a limit load solution for surface cracks in plates under biaxial force. This solution was extended to combined biaxial force and cross-thickness bending [7]. Those solutions were obtained from an uncracked plate under combined loading using the in-plane reference stress as an open-crack load to include the effect of the crack and are clearly not strict lower bound limit load solutions for the cracked plate. Recently, Madia et al. [8] developed a reference load equation for semi-elliptical surface cracks in plates under biaxial tension, based on detailed finite element (FE) *J* calculations for the deepest and surface points of the cracks. In this present paper, general lower bound limit load solutions for rectangular surface cracks in plates under combined biaxial tensile/compressive end force/stress and cross-thickness positive/negative bending are derived based on the lower bound limit load theorem [9] and the net-section collapse principle [10]. The limit load solutions obtained in this paper are

* Corresponding author.
E-mail address: yuebao.lei@edf-energy.com (Y. Lei).

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Nomenclature

a	crack depth
c	half crack length
C, D, E	parameters used to define limit load solutions
C_1, D_1, E_1	parameters used to define limit load solutions for combined parallel stress and bending moment
F_i, $i = 1,\ldots,4$	functions used in non-proportional loading solutions
J	crack driving force parameter
L	half plate length
M, M_L	applied and limit moment, respectively
m_L	normalised limit moment
m_{L0}, $m_{L\alpha}$, m_{L1}	m_L values for $\delta = 0$, α and 1, respectively
N, N_L	applied and limit end force, respectively
n_L	normalised limit end force
n_{L0}, $n_{L\alpha}$, n_{L1}	n_L values for $\delta = 0$, α and 1, respectively
n_{xL}	normalised limit x-direction stress, $=\sigma_{xL}/\sigma_y$
n_{xL1}, $n_{xL\alpha}$	n_{xL} values for $\delta = 1$ and α, respectively
R_a, R_b	parameters for limit load solutions due to Miura and Takahashi
$\overline{R}_b$	$\overline{R}_b = \lambda R_b$
S_{zc1}, S_{zc2}	normalised σ_{zc1} and σ_{zc2}, respectively
S_{zc11}	S_{zc1} value for $\delta = 1$
$S_{zc1\alpha}$, $S_{zc2\alpha}$	S_{zc1} and S_{zc2} values, respectively, for $\delta = \alpha$
t	plate thickness
W	half plate width
(x, y, z)	cartesian coordinates
$\overline{y}$	distance between neutral axis and the front surface of the plate
α	normalised crack depth, $=a/t$
β	normalised crack length, $=c/W$
δ	$=\overline{y}/t$
Φ	solution parameter for limit load solutions for $N \neq 0$
λ	ratio between M and Nt
λ_{01}, λ_{02}	characteristic load ratios used to define the valid region of solutions
λ_1	ratio between σ_x and σ_m
λ_2	ratio between σ_x and σ_b
λ_{201}, λ_{202}, λ_{203}	characteristic load ratios used to define the valid region of solutions for combined parallel stress and cross-thickness bending
σ_b	maximum elastic bending stress along the plate thickness
σ_{eqy}	reference stress for a surface crack in a plate under uniaxial membrane stress
σ_{eqyb}	reference stress for a surface crack in a plate under combined tension and positive bending
σ_m	membrane stress
σ_x,σ_{xL}	applied and limit membrane stress, respectively, parallel to crack plane (x-direction)
σ_y	yield stress
σ_z	z-direction stress
σ_{zc1}, σ_{zc2}	possible z-direction maximum and minimum stresses, respectively
Ω, Ω_1	parameter used to define λ_{02} and λ_{202}, respectively
ξ	solution parameter for limit load solutions for $N = 0$
Ψ	parameter for defining validity ranges of limit load solutions

based on the von Mises yield criterion. However, solutions for other yield criteria may be obtained by following the same methodology. The closed-form limit load solutions presented are for proportional loading. However, the basic equations are also valid for non-proportional loading and similar types of closed-form solutions for non-proportional loading can also be obtained.

The layout of this paper is as follows. Section 2 defines the geometry parameters and normalisations for the limit loads. Section 3 describes the derivation of limit load solutions. The limit load for the special case of combined bending moment and stress parallel to the crack plane is addressed in Section 4. The new limit load solutions are summarised and discussed in Section 5 and conclusions are given in Section 6. Appendix A contains details of the derivation of alternative validity ranges for the solutions. Appendix B gives a detailed proof for the correct choice of limit load expression from the various possible solutions of the governing equation. Appendix C provides the proof of the validity ranges of the limit load solutions for combined parallel stress and cross-thickness bending.

2. Parameter definitions

A general limit load solution procedure for a solid body containing a crack of arbitrary shape and under arbitrary loading conditions may be formulated based on the lower bound limit load theorem [9] without defining the details of the body and crack geometry. However, limit load solutions of this general type need to be further developed for use in practical engineering structural integrity assessments of particular components. In this paper, closed-form limit load solutions for plates containing a rectangular surface crack under combined loading are developed. Hence, the geometry parameters and loads need to be defined at the beginning of the work.

The geometry considered in this paper is a rectangular surface crack, with depth a and length $2c$, in a plate of width $2W$ and thickness t (Fig. 1). The loads considered are an end tensile/compressive force, N, a tensile/compressive membrane stress along the x-direction, σ_x, parallel to the crack plane, and a positive/negative cross-thickness bending moment, M, applied at the centroids of the end sections of the plate. The length of the plate, $2L$, is assumed to be large compared with the plate width and the thickness so that the St Venant principle may be applied at the crack plane. The limit loads corresponding to N, M and σ_x are represented by N_L, M_L and σ_{xL}, respectively, for an isotropic elastic-perfectly plastic material with a yield stress σ_y. Note that the "positive" directions for N, M and σ_x are shown in Fig. 1, where, a "positive" value of N or σ_x represents a tensile force/stress and a "negative" value represents a compressive force/stress. A "positive" value of M means that the moment tends to open the crack mouth whereas a "negative" moment tends to close the crack mouth.

The geometric parameters, normalised crack depth, α, and normalised crack length, β, are defined as follows, with $0 \le \alpha, \beta \le 1$.

$$\alpha = \frac{a}{t}, \beta = \frac{c}{W} \tag{1}$$

The normalised limit loads, n_L, m_L and n_{xL} for N, M and σ_x, respectively, are defined as

$$n_L = \frac{N_L}{2Wt\sigma_y},\ m_L = \frac{2M_L}{Wt^2\sigma_y},\ n_{xL} = \frac{\sigma_{xL}}{\sigma_y} \tag{2}$$

For proportional loading, the load ratio between the applied bending moment and end force along the z-direction, λ, is defined as